1) $\int \frac{dx}{\sin x \cos x}$

$$=\int \frac{2}{\sin 2x}dx = \int \csc 2x . \frac{\csc 2x+\cot 2x}{\csc 2x+\cot 2x} 2dx$$

$$=\int \frac{\csc^2 2x+\csc 2x\cot 2x}{\csc 2x+\cot 2x} 2dx = -\ln|\csc 2x + \cot 2x| + c$$

$$=\ln\left|\frac{1}{\csc 2x+\cot 2x}\right| + c$$

---

2) $\int_0^{\frac{\pi}{3}} \sec x\tan x dx$

$$=[\sec x]_0^{\frac{\pi}{3}} = \sec\frac{\pi}{3} - \sec 0 = 2 - 1 = 1$$

---

3) $\int \frac{\cos^4 x-\sin^4 x}{\sqrt{1+\cos 4x}} dx$

$$=\int \frac{(\cos^2 x-\sin^2 x)(\cos^2 x+\sin^2 x)}{\sqrt{1+2\cos^2 2x-1}} dx = \int \frac{\cos^2 x-\sin^2 x}{\sqrt{2\cos^2 2x}} dx$$

$$=\pm\frac{1}{\sqrt{2}}\int\frac{\cos 2x}{\cos 2x}dx = \pm\frac{1}{\sqrt{2}}\int dx = \pm\frac{1}{\sqrt{2}}x + c$$

---

4) $\int \frac{\tan\sqrt{x}}{\sqrt{x}} dx$

$$=\int \frac{\sin\sqrt{x}}{\cos\sqrt{x}}.\frac{1}{\sqrt{x}}dx = -2\int\frac{-\sin\sqrt{x}}{\cos\sqrt{x}}.\frac{1}{2\sqrt{x}}dx = -2\ln|\cos\sqrt{x}| + c$$

5) $\int \frac{2\csc^2 3x}{\sec 3x\sqrt{\sin 3x}}\,dx$

$= \int \frac{2.\frac{1}{\sin^2 3x}}{\frac{1}{\cos 3x}.\sin^{\frac{1}{2}} 3x}\,dx = 2\int \frac{1}{\sin^2 3x}.\frac{1}{\sin^{\frac{1}{2}} 3x}.\cos 3x\,dx$

$= 2\int \frac{\cos 3x}{\sin^{\frac{5}{2}} 3x}\,dx = 2\int \sin^{\frac{-5}{2}} 3x\cos 3x\,dx$

$= \frac{2}{3}.\frac{\sin^{\frac{-3}{2}} 3x}{\frac{-3}{2}} + c = -\frac{4}{9}\sin^{\frac{-3}{2}} 3x + c$

$= -\frac{4}{9}.\frac{1}{\sin^{\frac{3}{2}} 3x} + c = -\frac{4}{9}.\frac{1}{\sqrt{\sin^3 3x}} + c$

---

6) $\int \frac{e^{-\tan x}}{\cos^2 x}\,dx$

$= \int e^{-\tan x}.\frac{1}{\cos^2 x}\,dx = \int e^{-\tan x}\sec^2 x\,dx = -e^{-\tan x} + c$

---

7) $\int \frac{(\sin x-1)(\cos x+2)}{\cos x}\,dx$

$= \int \frac{(\sin x\cos x+2\sin x-\cos x-2)}{\cos x}\,dx = \int \left(\frac{\sin x\cos x}{\cos x} + 2\frac{\sin x}{\cos x} - \frac{\cos x}{\cos x} - \frac{2}{\cos x}\right)dx$

$= \int \left(\sin x + 2\frac{\sin x}{\cos x} - 1 - 2\sec x.\frac{\sec x+\tan x}{\sec x+\tan x}\right)dx$

$= \int \left(\sin x + 2\frac{\sin x}{\cos x} - 1 - 2\frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x}\right)dx$

$= -\cos x - 2\ln|\cos x| - x - 2\ln|\sec x + \tan x| + c$

$$8)\ \int \frac{\cos^4 x}{\sin^2 x}dx$$

$$=\int \frac{\cos^2 x\cos^2 x}{\sin^2 x}dx = \int \frac{\cos^2 x(1-\sin^2 x)}{\sin^2 x}dx = \int \frac{\cos^2 x-\cos^2 x\sin^2 x}{\sin^2 x}dx$$

$$=\int \left(\frac{\cos^2 x}{\sin^2 x} - \frac{\cos^2 x\sin^2 x}{\sin^2 x}\right)dx = \int \left(\frac{\cos^2 x}{\sin^2 x} - \cos^2 x\right)dx$$

$$=\int (\cot^2 x - \cos^2 x)dx = \int \left([\csc^2 x - 1] - \frac{1}{2}(1+\cos 2x\right)dx$$

$$=\int \left(\csc^2 x - \frac{3}{2} - \frac{1}{2}\cos 2x\right)dx$$

$$=-\cot x - \frac{3}{2}x - \frac{1}{4}\sin 2x + c$$

$$9)\ \int \frac{\cos^3 x}{\sin^2 x}dx$$

$$=\int \frac{\cos x(1-\sin^2 x)}{\sin^2 x}dx = \int \frac{\cos x-\cos x\sin^2 x}{\sin^2 x}dx$$

$$=\int \left(\frac{\cos x}{\sin^2 x} - \frac{\cos x\sin^2 x}{\sin^2 x}\right)dx = \int (\csc x\cot x - \cos x)dx$$

$$=-\csc x - \sin x + c$$

$$10)\ \int \frac{-\sin^2 x}{\cos x}dx$$

$$=\int \frac{-1+\cos^2 x}{\cos x}dx = \int \left(\frac{-1}{\cos x} + \frac{\cos^2 x}{\cos x}\right)dx$$

$$=\int \left(-\sec x.\frac{\sec x+\tan x}{\sec x+\tan x} + \cos x\right)dx = \int \left(-\frac{\sec^2 x+\sec x\tan x}{\sec x+\tan x} + \cos x\right)dx$$

$$=-\ln|\sec x + \tan x| + \sin x + c$$

11) $\int \frac{\tan^3 x}{\sqrt{\sec x}}dx$

$=\int \sec^{\frac{-1}{2}}x\tan x.\tan^2 x dx = \int \sec^{\frac{-1}{2}}x\tan x(\sec^2 x - 1)dx$

$=\int \left(\sec^{\frac{3}{2}}x\tan x - \sec^{\frac{-1}{2}}x\tan x\right)dx$

$=\int \left(sec^{\frac{1}{2}}x\ secxtanx - sec^{\frac{-3}{2}}x\ secxtanx\right)dx$

$=\frac{2}{3}\ sec^{\frac{3}{2}}x + 2sec^{\frac{-1}{2}}x + c$

$=\frac{2}{3}\sqrt{sec^3 x} + \frac{2}{\sqrt{secx}} + c$

---

12) $\int tan^3 3y\ sec^3 3ydy$

$=\int tan3y(sec^2 3y - 1)sec^3 3ydy$

$=\int (sec^4 3y\ sec3y\ tan3y - sec^2 3y\ sec3y\ tan3y)dy$

$=\frac{1}{3}.\frac{sec^5 3y}{5} - \frac{1}{3}.\frac{sec^3 3y}{3} + c = \frac{1}{15}sec^5 3y - \frac{1}{9}sec^3 3y + c$

---

13) $\int sin2x(sinx-4)^4(sinx+4)^4dx$

$=\int sin2x[(sinx-4)(sinx+4)]^4dx$

$=\int sin2x(sin^2-16)^4dx = \int (sin^2x-16)^4 2sinxcosxdx$

$=\frac{1}{5}(sin^2x-16)^5 + c$

14) $\int sinx(1-sinx)^3(1+sinx)^3dx$

$=\int sinx[(1-sinx)(1+sinx)]^3dx$

$=\int sinx(1-sin^2x)^3dx=\int sinx(cos^2x)^3dx$

$=\int cos^6x\ sinx\ dx=-\frac{1}{7}cos^7x+c$

---

15) $\int \frac{cos^3x-4}{cosx}dx$

$=\int \left(\frac{cos^3x}{cosx}-\frac{4}{cosx}\right)dx=\int(cos^2x-4secx)dx$

$=\int \left[\left(\frac{1}{2}+\frac{1}{2}cos2x\right)-4secx.\frac{secx+tanx}{secx+tanx}\right]dx$

$=\frac{1}{2}x+\frac{1}{4}sin2x-4ln|secx+tanx|+c$

---

16) $\int \frac{dx}{csc2x-cot2x}$

$=\int \frac{1}{\frac{1}{sin2x}-\frac{cos2x}{sin2x}}dx=\int \frac{sin2x}{1-cos2x}dx$

$=\frac{1}{2}\int \frac{2sin2x}{1-cos2x}dx=\frac{1}{2}ln|1-cos2x|+c$

$=ln\left|\sqrt{1-cos2x}\right|+c$

17) $\int \frac{1}{1+cos3x}dx$

$=\int \frac{1}{1+cos3x}.\frac{1-cos3x}{1-cos3x}dx = \int \frac{1-cos3x}{1-cos^2 3x}dx = \int \frac{1-cos3x}{sin^2 3x}dx$

$=\int \left(\frac{1}{sin^2 3x} - \frac{cos3x}{sin^2 3x}\right)dx = \int (csc^2 3x - csc3xcot3x)dx$

$= -\frac{1}{3}cot3x + \frac{1}{3}csc3x + c$

---

18) $\int (cosx - sin2x)^2 dx$

$=\int (cosx - 2sinxcosx)^2 dx = \int cos^2 x(1-2sinx)^2 dx$

$=\int cos^2 x(1-4sinx+4sin^2 x)dx$

$=\int (cos^2 x - 4cos^2 x\, sinx + 4sin^2 xcos^2 x)dx$

$=\int \left[\frac{1}{2}+\frac{1}{2}cos2x - 4cos^2 x\, sinx + (2sinx\, cosx)^2\right]dx$

$=\int \left[\frac{1}{2}+\frac{1}{2}cos2x - 4cos^2 x\, sinx + sin^2 2x\right]dx$

$=\int \left[\frac{1}{2}+\frac{1}{2}cos2x - 4cos^2 x\, sinx + \frac{1}{2} - \frac{1}{2}cos4x\right]dx$

$=x + \frac{1}{4}sin2x + \frac{4}{3}cos^3 x - \frac{1}{8}sin4x + c$

---

19) $\int sec^{\frac{1}{2}}x\, tanxdx$

$=\int sec^{\frac{-1}{2}}x\, secx\, tanx\, dx = 2sec^{\frac{1}{2}}x + c = 2\sqrt{secx} + c$

20) $\int \frac{tan2x}{\sqrt{cos2x}} dx$

$=\int \frac{1}{\sqrt{cos2x}}\ tan2xdx = \int \sqrt{sec2x}\, tan2xdx$

$=\int sec^{\frac{-1}{2}}2x\ sec2x\ tan2xdx = \frac{1}{2}.2sec^{\frac{1}{2}}2x + c = \sqrt{sec2x} + c$

---

٢١) $\int \frac{sin^4x}{sinx+\frac{1}{2}sin2x} dx$

$=\int \frac{sin^4x}{sinx+sinxcosx} dx = \int \frac{sin^4x}{sinx(1+cosx)} dx = \int \frac{sin^3x}{1+cosx} dx$

$=\int \frac{sin^2xsinx}{1+cosx} dx = \int \frac{(1-cos^2x)sinx}{1+cosx} dx$

$=\int \frac{(1+cosx)(1-cosx)sinx}{1+cosx} dx = \int (1-cosx)sinxdx$

$=\int (sinx - sinxcosx)dx = -cosx - \frac{1}{2}sin^2x + c$

---

22) $\int_{\frac{-\pi}{4}}^{\frac{\pi}{3}} sin^2x\left(1 + cot^2x\right)dx$

$=\int_{\frac{-\pi}{4}}^{\frac{\pi}{3}} sin^2x.csc^2xdx = \int_{\frac{-\pi}{4}}^{\frac{\pi}{3}} \frac{sin^2x}{sin^2x} dx = \int_{\frac{-\pi}{4}}^{\frac{\pi}{3}} dx$

$=[x]_{\frac{-\pi}{4}}^{\frac{\pi}{3}} = \frac{\pi}{3} + \frac{\pi}{4} = \frac{7\pi}{12}$

23) $\int cos^3x dx$

$=\int cos^2x\, cosx dx = \int (1 - sin^2x) cosx dx$

$=\int (cosx - sin^2x cosx) dx = sinx - \frac{1}{3} sin^3x + c$

---

24) $\int tan^3x\, sec^4x\, dx$

$=\int tan^3x\, sec^2x\, .\, sec^2x dx = \int tan^3x\, sec^2x (tan^2x + 1) dx$

$=\int \left(tan^5x\, sec^2x + tan^3x\, sec^2x\right) dx$

$=\frac{1}{6} tan^6x + \frac{1}{4} tan^4x + c$

---

25) $\int tan^3x\, sec^3x dx$

$=\int tan^2x\, tanx\, sec^3x\, dx = \int sec^3x\, tanx (sec^2x - 1) dx$

$=\int (sec^4x\, secx tanx - sec^2x\, secx\, tanx) dx$

$=\frac{1}{5} sec^5x - \frac{1}{3} sec^3x + c$

---

26) $\int x^7 tan\left(8x^8 + 6\right) dx$

$=-\frac{1}{64}\int \frac{sin\left(8x^8+6\right)}{cos\left(8x^8+6\right)}(-64x^7) dx = -\frac{1}{64} ln\left|cos(8x^8 + 6)\right| + c$

---

27) $\int \frac{cos(3+5ln9x)}{7x} dx$

$=\frac{1}{7}\int cos(3 + 5ln9x) . \frac{1}{x} dx = \frac{1}{35} sin(3 + 5ln9x) + c$

28) $\int cot^3x\, csc^5x dx$

$=\int (csc^2x - 1)cotx csc^5x\, dx = \int (csc^7x cotx - csc^5x\, cotx\,)\, dx$

$=\int (csc^6x\, cscx\, cotx - csc^4x\, cscx\, cotx\,)dx$

$=-\frac{1}{7}csc^7x + \frac{1}{5}csc^5x + c$

---

29) $\int \frac{\sqrt[3]{sinx}}{cos^{\frac{7}{3}}x}dx$

$=\int \frac{sin^{\frac{1}{3}}x}{cos^{\frac{1}{3}}x}\cdot\frac{1}{cos^2x}dx = \int \left(\frac{sinx}{cosx}\right)^{\frac{1}{3}}\cdot\frac{1}{cos^2x}dx = \int tan^{\frac{1}{3}}x\, sec^2x\, dx$

$=\frac{3}{4}tan^{\frac{4}{3}}x + c = \frac{3}{4}\sqrt[3]{tan^4x} + c$

---

30) $\int \frac{dx}{sinx}$

$=\int cscx.\frac{cscx+cotx}{cscx+cotx}dx = \int \frac{csc^2x+cscx\, cotx}{cscx+cotx}dx = -ln|cscx + cotx| + c$

---

31) $\int \frac{1}{cos^2x}dx$

$=\int sec^2xdx = tanx + c$

32) $\int sin^3 3xdx$

$=\int sin^2 3x.\, sin3xdx = \int (1 - cos^2 3x)sin3x\, dx$

$=\int (sin3x - cos^2 3x\, sin3x)dx$

$=-\frac{1}{3}cos3x + \frac{1}{9}cos^3 3x + c$

33) $\int sec^4 3x\,dx$

$=\int sec^2 3x sec^2 3x\,dx = \int sec^2 3x(tan^2 3x + 1)dx$

$=\int (tan^2 3x sec^2 3x + sec^2 3x)dx$

$=\frac{1}{3}\frac{tan^3 3x}{3} + \frac{1}{3}tan3x + c = \frac{1}{9}tan^3 3x + \frac{1}{3}tan3x + c$

....

34) $\int tan^3 2x\,dx$

$=\int tan^2 2x\,tan2x\,dx = \int (sec^2 2x - 1)tan2x\,dx$

$=\int (tan2x\,sec^2 2x - tan2x)dx$

$=\int \left(tan2x\,sec^2 2x - \frac{sin2x}{cos2x}\right)dx$

$=\frac{1}{4}tan^2 2x + \frac{1}{2}ln|cos2x| + c$

....

35) $\int x\,cos\left(x^2 - 4\right)dx$

$=\frac{1}{2}sin(x^2 - 4) + c$

....

36) $\int \sqrt[3]{(sinx - cosx)(1 - sin2x)}dx$

$= \int \sqrt[3]{(sinx - cosx)(sin^2 x + cos^2 x - 2sinxcosx)}dx$

$= \int \sqrt[3]{(sinx - cosx)(sinx - cosx)^2}dx = \int \sqrt[3]{(sinx - cosx)^3}dx$

$= \int (sinx - cosx)dx = -cosx - sinx + c$

OR

$$\int \sqrt[3]{(sinx - cosx)(1 - sin2x)}dx$$

$$= \int \sqrt[3]{(sinx - cosx)(sin^2x + cos^2x - 2sinxcosx)}dx$$

$$\int \sqrt[3]{-(cosx - sinx)(cosx - sinx)^2}dx = \int \sqrt[3]{-(cosx - sinx)^3}dx$$

$$= -\int (cosx - sinx)\mathrm{d}x = -[sinx + cosx] + c$$

$$= -sinx - cosx + c$$

---

37) $\int \sqrt[5]{csc^{11}x}.\sqrt[5]{cosx}dx$

$$=\int \sqrt[5]{csc^{10}x}.\sqrt[5]{\frac{cosx}{sinx}}dx = \int \sqrt[5]{cotx}.\sqrt[5]{(csc^2)^5}dx$$

$$=\int (cotx)^{\frac{1}{5}}csc^2x\,dx = -\frac{cot^{\frac{6}{5}}x}{\frac{6}{5}} + c$$

$$=-\frac{5}{6}\sqrt[5]{cot^6x} + c$$

---

38) $\int \frac{sin2x}{\sqrt[3]{5+4sin^2x}}dx$

$$=\int \frac{sin2x}{(5+4sin^2x)^{\frac{1}{3}}}dx = \int (5 + 4sin^2x)^{\frac{-1}{3}}(2sinx\,cosx)dx$$

$$=\frac{1}{4}.\frac{3}{2}(5 + sin^2x)^{\frac{2}{3}} + c = \frac{3}{8}\sqrt[3]{(5 + 4sin^2x)^2} + c$$

39) $\int \frac{sin8x}{(2-cos8x)^2} dx$

$= \int (2-cos8x)^{-2} sin8x\, dx = \frac{1}{8} \frac{(2-cos8x)^{-1}}{-1} + c$

$= \frac{-1}{8(2-cos8x)} + c$

---

40) $\int \sqrt[3]{tan2x+5} sec^2 2x\, dx$

$= \int (tan2x+5)^{\frac{1}{3}} sec^2 2x dx = \frac{1}{2} . \frac{3}{4} (tan2x+5)^{\frac{4}{3}} + c$

$= \frac{3}{8} \sqrt[3]{(tan2x+5)^4} + c$

---

٤١) $\int sin5x\, cos5x\, dx$

$= \frac{1}{5} . \frac{1}{2} sin^2 5x + c = \frac{1}{10} sin^2 5x + c$

OR : $-\frac{1}{10} cos^2 5x + c$

---

٤٢) $\int \frac{cosx}{(2+sinx)^2} dx$

$= \int (2+sinx)^{-2} cosx\, dx = \frac{(2+sinx)^{-1}}{-1} + c$

$= -\frac{1}{2+sinx} + c$

٤٣) $\int \frac{\sin 2x}{\cos^2 2x} dx$

$= \int \frac{\sin 2x}{\cos 2x} . \frac{1}{\cos 2x} dx = \int \tan 2x . \sec 2x dx = \frac{1}{2} \sec 2x + c$

OR: $\int \frac{\sin 2x}{\cos^2 2x} dx = \int (\cos 2x)^{-2} \sin 2x dx = -\frac{1}{2} . \frac{(\cos 2x)^{-1}}{-1} + c$

$= \frac{1}{2} . \frac{1}{\cos 2x} + c = \frac{1}{2} \sec 2x + c$

....

44) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (-\csc^2 x e^{\cot x}) dx$

$= [e^{\cot x}]_{\frac{\pi}{4}}^{\frac{\pi}{2}} = e^{\cot\frac{\pi}{2}} - e^{\cot\frac{\pi}{4}} = e^0 - e^1 = 1 - e$

....

45) $\int (\sec x \tan x - 1)^2 dx$

$= \int (\tan^2 x \sec^2 x - 2\sec x \tan x + 1) dx$

$= \frac{1}{3} \tan^3 x - 2\sec x + x + c$

....

46) $\int \sin 3x dx$

$= \frac{-1}{3} \cos 3x + c$

....

47) $\int \cos(3x - 1) dx$

$= \frac{1}{3} \sin(3x - 1) + c$

48) $\int tan^3 x dx$

$= \int tan^2 x\, tanx\, dx = \int (sec^2 x - 1) tanx\, dx$

$= \int (tanx\, sec^2 x - tanx) dx = \frac{1}{2} tan^2 x + ln|cosx| + c$

....

49) $\int csc^4 x dx$

$= \int csc^2 c\, csc^2 x\, dx = \int csc^2 x (cot^2 x + 1) dx$

$= \int (cot^2 x\, csc^2 x + csc^2 x\,) dx = -\frac{1}{3} cot^3 x - cotx + c$

....

50) $\int sin(sinx) cosx\, dx$

$= -cos(sinx) + c$

....

51) $\int \sqrt{secx} secx\, tanx\, dx$

$= \int (secx)^{\frac{1}{2}} secx\, tanx\, dx = \frac{2}{3} sec^{\frac{3}{2}} x + c = \frac{2}{3} \sqrt{sec^3 x} + c$

....

52) $\int \frac{dx}{1 - cosx}$

$= \int \frac{1}{1 - cosx} . \frac{1 + cosx}{1 + cosx} dx = \int \frac{1 + cosx}{1 - cos^2 x} dx = \int \frac{1 + cosx}{sin^2 x} dx$

$= \int (csc^2 x + cscx\, cotx) dx = -cotx - cscx + c$

53) $\int \frac{dx}{1+sinx}$

$= \int \frac{1}{1+sinx} \cdot \frac{1-sinx}{1-sinx} dx = \int \frac{1-sinx}{1-sin^2x} dx = \int \frac{1-sinx}{cos^2x} dx$

$= \int (sec^2x - secx\, tanx)dx = tanx - secx + c$

---

54) $\int cos^8x sin^3x dx$

$= \int cos^8x sin^2x\, sinx\, dx = \int cos^8x\, sinx(1 - cos^2x)dx$

$= \int (cos^8x\, sinx - cos^{10}x\, sinx\,)\, dx$

$= -\frac{1}{9} cos^9x + \frac{1}{11} cos^{11}x + c$

---

55) $\int secx\, csc^2xdx$

$= \int secx\, (cot^2x + 1)dx = \int (secx\, cot^2x + secx)dx$

$= \int \left(\frac{cos^2x}{sin^2x} \cdot \frac{1}{cosx} + secx\right) dx = \int \left(\frac{cosx}{sinx} \cdot \frac{1}{sins} + secx\right) dx$

$= \int (cscx\, cotx + secx)dx = -cscx + ln|secx + tanx| + c$

---

56) $\int \frac{sinxcosx}{3+2cos^2x} dx$

$= -\frac{1}{4} ln\left|3 + 2cos^2x\right| + c$

57) $\int \frac{cosx-sinx}{(1+sin2x)^8}dx$

$=\int \frac{cosx-sinx}{\left(sin^2x+2sinx\,cosx+cos^2x\right)^8}dx = \int \frac{cosx-sinx}{\left((sinx+cosx)^2\right)^8}dx$

$=\int \frac{cosx-sinx}{(sinx+cosx)^{16}}dx = \int (sinx+cosx)^{-16}(cosx-sinx)dx$

$=-\frac{1}{15}(sinx+cosx)^{-15}+c = -\frac{1}{15(sinx+cosx)^{15}}+c$

58) $\int\left[cot2x\sqrt{sin2x}+cos2x\right]dx$

$=\int\left[\frac{cos2x}{sin2x}.(sin2x)^{\frac{1}{2}}+cos2x\right]dx$

$=\int\left[(sin2x)^{-1}(sin2x)^{\frac{1}{2}}cos2x+cos2x\right]dx$

$=\int\left[(sin2x)^{\frac{-1}{2}}cos2x+cos2x\right]dx$

$=\frac{1}{2}.2(sin2x)^{\frac{1}{2}}+\frac{1}{2}sin2x+c$

$=\sqrt{sin2x}+\frac{1}{2}sin2x+c$

59) $\int_0^{\frac{3\pi}{4}}|cosx|dx$

$=cosx=0 \rightarrow x=\frac{\pi}{2}\in\left[0,\frac{3\pi}{4}\right]$

$=\int_0^{\frac{\pi}{2}}cosx\,dx+\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}}cosx\,dx=[sinx]_0^{\frac{\pi}{2}}+[sinx]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$

$=\left[sin\frac{\pi}{2}-sin0\right]+\left[sin\frac{3\pi}{4}-sin\frac{\pi}{2}\right]=(1-0)+\left(\frac{1}{\sqrt{2}}-1\right)=\frac{1}{\sqrt{2}}$

60) $\int \frac{1-\sin 4x}{\sin 2x-\cos 2x}\,dx$

$$=\int \frac{\sin^2 2x+\cos^2 2x-2\sin 2x\cos 2x}{\sin 2x-\cos 2x}\,dx$$

$$=\int \frac{\sin^2 2x-2\sin 2x\cos 2x+\cos^2 2x}{\sin 2x-\cos 2x}\,dx$$

$$=\int \frac{(\sin 2x-\cos 2x)^2}{\sin 2x-\cos 2x}\,dx=\int(\sin 2x-\cos 2x)\,dx$$

$$=-\frac{1}{2}\cos 2x-\frac{1}{2}\sin 2x+c$$

---

61) $\int \frac{\sin^3 3x}{\cos 3x-1}\,dx$

$$=\int \frac{\sin^2 3x\,\sin 3x}{\cos 3x-1}\,dx=\int \frac{(1-\cos^2 3x)\sin 3x}{\cos 3x-1}\,dx$$

$$=\int \frac{(1-\cos 3x)(1+\cos 3x)\sin 3x}{-(1-\cos 3x)}\,dx=-\int(\sin 3x+\sin 3x\cos 3x)\,dx$$

$$=-\left[-\frac{1}{3}\cos 3x+\frac{1}{6}\sin^2 3x\right]+c=\frac{1}{3}\cos 3x-\frac{1}{6}\sin^2 3x+c$$

---

62) $\int \frac{1}{\cos^2 x\sin^2 x}\,dx$

$$=\int \frac{\cos^2 x+\sin^2 x}{\cos^2 x\sin^2 x}\,dx=\int\left(\frac{\cos^2 x}{\cos^2 x\sin^2 x}+\frac{\sin^2 x}{\cos^2 x\sin^2 x}\right)dx$$

$$=\int\left(\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}\right)dx=\int(\csc^2 x+\sec^2 x)\,dx$$

$$=-\cot x+\tan x+c$$

63) $\int \sin^4 x \cos^2 x \, dx$

$$=\int \left[\left(\frac{1}{2}(1-\cos 2x)\right)^2\left(\frac{1}{2}(1+\cos 2x)\right)\right]dx$$

$$=\frac{1}{8}\int\left[\left(1-2\cos 2x+\frac{1}{2}+\frac{1}{2}\cos 4x\right)(1+\cos 2x)\right]dx$$

$$=\frac{1}{8}\int\left[\left(\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x\right)(1+\cos 2x)\right]dx$$

$$=\frac{1}{8}\int\left[\frac{3}{2}-2\cos 2x+\frac{1}{2}\cos 4x+\frac{3}{2}\cos 2x-2\cos^2 2x+\frac{1}{2}\cos 2x\cos 4x\right]dx$$

$$=\frac{1}{8}\int\left[\frac{3}{2}-\frac{1}{2}\cos 2x+\frac{1}{2}\cos 4x-1-\cos 4x+\frac{1}{2}(1-2\sin^2 2x)\cos 2x\right]dx$$

$$=\frac{1}{8}\int\left[\frac{1}{2}-\frac{1}{2}\cos 2x-\frac{1}{2}\cos 4x+\frac{1}{2}\cos 2x-\sin^2 2x\cos 2x\right]dx$$

$$=\frac{1}{8}\int\left[\frac{1}{2}-\frac{1}{2}\cos 4x-\sin^2 2x\cos 2x\right]dx$$

$$=\frac{1}{8}\left[\frac{1}{2}x-\frac{1}{8}\sin 4x-\frac{1}{6}\sin^3 2x\right]+c$$

$$=\frac{x}{16}-\frac{1}{64}\sin 4x-\frac{1}{48}\sin^3 2x+c$$

....

64) $\int \frac{\cos 2x}{\sin x+\cos x}dx$

$$=\int\frac{\cos^2 x-\sin^2 x}{\sin x+\cos x}dx=\int\frac{(\cos x-\sin x)(\cos x+\sin x)}{\sin x+\cos x}dx$$

$$=\int(\cos x-\sin x)dx=\sin x+\cos x+c$$

65) $\int \frac{cos2x}{2csc^2x}dx + \int \frac{sin2x}{2\,sec^2x}dx$

$=\int \frac{2cos^2x-1}{2csc^2x}dx + \int \frac{2sinx\,cosx}{2sec^2x}dx$

$=\int \left(\frac{2cos^2x}{2csc^2x} - \frac{1}{2csc^2x}\right)dx + \int (cos^3x\,sinx)dx$

$=\int \left[\left(\frac{2}{2}sin^2xcos^2x\right) - \left(\frac{1}{2}sin^2x\right)\right]dx + \int (cos^3x\,sinx)dx$

$=\int \left[\frac{1}{4}sin^2 2x - \frac{1}{2}sin^2x\right]dx + \int (cos^3x\,sinx)dx$

$=\int \left[\frac{1}{4}\left(\frac{1}{2} - \frac{1}{2}cos4x\right) - \frac{1}{2}\left(\frac{1}{2} - \frac{1}{2}cos2x\right)\right]dx + \int (cos^3x\,sinx)dx$

$=\int \left[\frac{1}{8} - \frac{1}{8}cos4x - \frac{1}{4} + \frac{1}{4}cos2x\right]dx + \int (cos^3x\,sinx)dx$

$=\int \left(-\frac{1}{4} - \frac{1}{8}cos4x + \frac{1}{4}cos2x\right)dx + \int (cos^3x\,sinx)dx$

$=-\frac{1}{4}x - \frac{1}{32}sin4x + \frac{1}{8}sin2x - \frac{1}{4}cos^4x + c$

---

66) $\int cos^{-4}x\,dx$

$=\int sec^4x\,dx = \int sec^2x(tan^2x + 1)dx$

$=\int (tan^2x\,sec^2x + sec^2x)dx$

$=\frac{1}{3}tan^3x + tanx + c$

67) $\int \frac{1-\cos 2x}{\sin x(1+\cos 2x)}\,dx$

$=\int \frac{1-(1-2\sin^2 x)}{\sin x(1+(2\cos^2 x-1))}\,dx = \int \frac{2\sin^2 x}{\sin x(2\cos^2 x)}\,dx$

$=\int \frac{\sin x}{\cos^2 x}\,dx = \int \sec x \tan x\,dx = \sec x + c$

---

68) $\int \frac{\cos 5x}{e^{\sin 5x}}\,dx$

$=\int \left(e^{\sin 5x}\right)^{-1} \cos 5x\,dx = \int e^{-\sin 5x}\cos 5x\,dx$

$=-\frac{1}{5}e^{-\sin 5x} + c = \frac{-1}{5e^{\sin 5x}} + c$

---

69) $\int \frac{4\csc^2 2x}{\sqrt{1-3\cot 2x}}\,dx$

$=-\frac{4}{6}\int (1-3\cot 2x)^{\frac{-1}{2}}(-6\csc^2 2x)dx$

$=-\frac{2}{3}.\,2(1-3\cot 2x)^{\frac{1}{2}} + c = -\frac{4}{3}\sqrt{1-3\cot 2x} + c$

---

70) $\int \frac{e^x \sec e^x \tan e^x}{\sec e^x + e}\,dx$

$=\ln|\sec e^x + e| + c$

---

71) $\int \frac{2\sin\sqrt[3]{x}}{\sqrt[3]{x^2}}\,dx$

$=2\int \sin x^{\frac{1}{3}}.\,x^{\frac{-2}{3}}dx = -6\cos\sqrt[3]{x} + c$

72) $\int (cosx - sinx)^4 cos2x\ dx$

$=\int (cos^2x - 2sinxcosx + sin^2x)^2 cos2x\ dx$

$=\int (1 - sin2x)^2 cos2x\ dx = \int (1 - 2sin2x + sin^2x) cos2x\ dx$

$=\int (cos2x - 2sin2xcos2x + sin^2 2x\ cos2x) dx$

$=\frac{1}{2} sin2x - \frac{1}{2} sin^2 2x + \frac{1}{6} sin^3 2x + c$

OR: $\int (cosx - sinx)^4 cos2x\ dx$

$=\int (cos^2x - 2sinxcosx + sin^2x)^2 cos2x\ dx$

$=\int (1 - sin2x)^2 cos2x\ dx = \int (1 - 2sin2x + sin^2x) cos2x\ dx$

$=\int (cos2x - sin4x + sin^2 2x\ cos2x) dx$

$=\frac{1}{2} sin2x + \frac{1}{4} cos4x + \frac{1}{6} sin^3 2x + c$

OR: $\int (cosx - sinx)^4 cos2x\ dx$

$=\int (cos^2x - 2sinxcosx + sin^2x)^2 cos2x\ dx$

$=\int (1 - sin2x)^2 cos2x\ dx = -\frac{1}{2} . \frac{1}{3} (1 - sin2x)^3 + c$

$=-\frac{1}{6} (1 - sin2x)^3 + c$

73) $\int sin4x\ cscx\ dx$

$=\int 2sin2x\ cos2x\ cscx\ dx = \int 4sinx\ cosx\ cos2x . \frac{1}{sinx} dx$

$=4 \int cosx\ cos2x\ dx = 4 \int cosx(1 - 2sin^2x) dx$

$=4 \int (cosx - 2sin^2xcosx) dx = 4 \left[sinx - \frac{2}{3} sin^3x \right] + c$

$=4sinx - \frac{8}{3} sin^3x + c$

74) $\int sec^2x csc^2x\, dx$

$=\int (tan^2x+1)csc^2x\, dx = \int (tan^2x csc^2x + csc^2x)dx$

$=\int \left(\frac{sin^2x}{cos^2x}\frac{1}{sin^2x} + csc^2x\right)dx = \int (sec^2x + csc^2x)dx$

$=tanx - cotx + c$

OR: $\int sec^2x csc^2x\, dx = \int \frac{1}{sin^2x cos^2x}dx$

$=\int \frac{4}{4sin^2x cos^2x}dx = 4\int \frac{1}{(2sinx\, cosx)^2}dx$

$=4\int \frac{1}{sin^2 2x}dx = 4\int csc^2 2x\, dx = -2\, cot2x + c$

---

75) $\int \frac{1-cos2x}{1+cos2x}dx$

$=\int \frac{1-(1-2sin^2x)}{1+2cos^2x-1}dx = \int \frac{2sin^2x}{2cos^2x}dx = \int tan^2x\, dx$

$=\int (sec^2x - 1)dx = tanx - x + c$

OR: $\int \frac{1-cos2x}{1+cos2x}dx = \int \frac{1-cos2x}{1+cos2x}\cdot\frac{1-cos2x}{1-cos2x}dx$

$=\int \frac{1-2cos2x+cos^2 2x}{1-cos^2 2x}dx = \int \left(\frac{1}{sin^2 2x} - \frac{2cos2x}{sin^2 2x} + \frac{cos^2 2x}{sin^2 2x}\right)dx$

$=\int (csc^2 2x - 2csc2x\, cot2x + cot^2 2x)dx$

$=\int (csc^2 2x - 2csc2x\, cot2x + csc^2 2x - 1)dx$

$=\int (2csc^2 2x - 2csc2x\, cot2x - 1)dx$

=-cot2x+csc2x-x+c

76) $\int \frac{sin2x}{sin^3x}dx$

$= \int \frac{2\,sinxcosx}{sin^3x}dx = 2\int \frac{cosx}{sin^2x}dx = 2\int cscx\,cotx = -2cscx + c$

....

77) $\int \frac{sin2x}{sin^2x}dx$

$=\int \frac{2\,sinxcosx}{sin^2x}dx = 2\int \frac{cosx}{sinx}dx = 2ln|sinx| + c$

....

78) $\int sin^3xcos^3x\,dx$

$=\int sin^2x\,sinxcos^3xdx = \int cos^3x\,sinx(1-cos^2x)dx$

$=\int\left(cos^3x\,sinx - cos^5x\,sinx\right)dx = -\frac{1}{4}cos^4x + \frac{1}{6}cos^6x + c$

OR: $\int sin^3xcos^3x\,dx = \int sin^3xcos^2x\,cosxdx$

$=\int\left(sin^3xcosx - sin^5x\,cosx\right)dx = \frac{1}{4}sin^4x - \frac{1}{6}sin^6x + c$

OR: $\int sin^3xcos^3x\,dx = \int (sinx\,cosx)^3dx$

$=\frac{1}{8}\int sin^32xdx = \frac{1}{8}\int(sin2x - cos^22x\,sin2x)dx$

$=\frac{1}{8}\left[-\frac{1}{2}cos2x + \frac{1}{6}cos^32x\right] + c = -\frac{1}{16}cos2x + \frac{1}{48}cos^32x + c$

79) $\int \frac{1-tanx}{1+tanx}dx$

$=\int \frac{\frac{cosx-sinx}{cosx}}{\frac{cosx+sinx}{cosx}}dx = \int \frac{cosx-sinx}{cosx+sinx}dx = ln|cosx + sinx| + c$

OR: $\int \frac{1-tanx}{1+tanx}dx = \int \frac{\frac{cosx-sinx}{cosx}}{\frac{cosx+sinx}{cosx}}dx = \int \frac{cosx-sinx}{cosx+sinx}.\frac{cosx-sinx}{cosx-sinx}dx$

$=\int \frac{cos^2x-2sinxcosx+sin^2x}{cos^2x-sin^2x}dx = \int \frac{1-sin2x}{cos2x}dx$

$=\int (sec2x)dx - \int \frac{sin2x}{cos2x}dx$

$= \frac{1}{2}ln|sec2x + tan2x| + \frac{1}{2}ln|cos2x| + c$

---

80) $\int sin2x\ cotx\ dx$

$=\int 2sinx\ cosx.\frac{cosx}{sinx}dx = 2\int cos^2xdx$

$=2\int \frac{1}{2}(1 + cos2x)dx = x + \frac{1}{2}sin2x + c$

---

81) $\int sin6x\ cos^2 3x\ dx$

$=\int 2sin3x\ cos3x\ cos^2 3xdx = 2\int cos^3 3x\ sin3x\ dx$

$=-\frac{2}{3}.\frac{1}{4}cos^4 3x + c = -\frac{1}{6}cos^4 3x + c$

82) $\int \frac{1}{(cosx-sinx)^2(1+sin2x)}dx$

$=\int \frac{1}{(cosx-sinx)^2(cos^2x+2sinx\,cosx+sin^2x)}dx$

$=\int \frac{1}{(cosx-sinx)^2(cosx+sinx)^2}dx = \int \frac{1}{[(cosx-sinx)(cosx+sinx)]^2}dx$

$=\int \frac{1}{\left(cos^2x-sin^2x\right)^2}dx = \int \frac{1}{cos^2 2x}dx = \int sec^2 2x dx$

$=\frac{1}{2}\,tan2x + c$

---

83) $\int \frac{tan^3x sec^2x}{sec^4x-1}dx$

$=\int \frac{(sec^2x-1)tanx sec^2x}{(sec^2x-1)(sec^2x+1)}dx = \frac{1}{2}\int \frac{2tanx\,sec^2x}{sec^2x+1}dx$

$=\frac{1}{2}\,ln\left|sec^2x + 1\right| + c$

---

84) $\int \frac{2sin^2x+cos2x}{sec^2x}dx$

$=\int \frac{2sin^2x+cos^2x-sin^2x}{sec^2x}dx = \int \frac{sin^2x+cos^2x}{sec^2x}dx$

$=\int \frac{1}{sec^2x}dx = \int cos^2x dx = \frac{1}{2}\int(1 + cos2x)dx$

$=\frac{1}{2}\left(x + \frac{1}{2}sin2x\right) + c = \frac{1}{2}x + \frac{1}{4}sin2x + c$

85) $\int \frac{sec^5x\, tanx}{\sqrt{cosx}}dx$

$=\int sec^5x\, tanx\, sec^{\frac{1}{2}}x\, dx = \int sec^{\frac{11}{2}}x\, tanx\, dx =$

$\int sec^{\frac{9}{2}}x\, secx\, tanx\, dx$

$=\frac{2}{11}sec^{\frac{11}{2}}x + c = \frac{2}{11}\sqrt{sec^{11}x} + c$

....

86) $\int \frac{1-cos2x}{sin2x}dx$

$=\int \left(\frac{1}{sin2x} - \frac{cos2x}{sin2x}\right)dx = \int \left(csc2x.\frac{csc2x+cot2x}{csc2x+cot2x} - \frac{cos2x}{sin2x}\right)dx$

$=-\frac{1}{2}ln|csc2x + cot2x| - \frac{1}{2}ln|sin2x| + c$

....

87) $\int \frac{sinx+cosx}{cos^3x}dx$

$=\int \left(\frac{sinx}{cos^3x} + \frac{cosx}{cos^3x}\right)dx = \int (tanxsec^2x + sec^2x)dx$

$=\frac{1}{2}tan^2x + tanx + c$

....

88) $\int \frac{e^{tanx}+sin^2x e^{tanx}}{sin^4x-1}dx$

$=\int \frac{e^{tanx}(1+sin^2x)}{(sin^2x-1)(sin^2x+1)}dx = -\int \frac{e^{tanx}}{cos^2x}dx$

$=-\int e^{tanx}sec^2xdx = -e^{tanx} + c$

89) $\int tan^5x\,dx$

$=\int tan^3x tan^2x\,dx = \int tan^3x\,(sec^2x-1)dx$

$=\int \left(tan^3x sec^2x - tanx(sec^2x-1)\right)dx$

$=\int (tan^3x sec^2x - tanx sec^2x + tanx)dx$

$=\int \left(tan^3x sec^2x - tanx sec^2 + \frac{sinx}{cosx}\right)dx$

$=\frac{1}{4}tan^4x - \frac{1}{2}tan^2x - ln|cosx| + c$

---

90) $\int sec^5x\,dx$

$=\int sec^n x dx = \frac{1}{n}tanx\,sec^{n-2}x + \frac{n-2}{n-1}\int sec^{n-2}x dx$

$=\frac{1}{4}tanx sec^3x + \frac{3}{4}\int sec^3x dx$

$=\frac{1}{4}tanx\,sec^3x + \frac{3}{4}(\frac{1}{2}tanx\,secx + \frac{1}{2}\int secx dx)$

$=\frac{1}{4}tanx\,sec^3x + \frac{3}{8}tanx\,secx + \frac{3}{8}ln|secx + tanx| + c$

---

91) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}(tanx + tan^3x)dx$

$=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tanx(1 + tan^2x)dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} tanx sec^2x\,dx$

$=\left[\frac{1}{2}tan^2x\right]_{\frac{\pi}{6}}^{\frac{\pi}{3}} = \frac{1}{2}\left[\left(tan\frac{\pi}{3}\right)^2 - \left(tan\frac{\pi}{6}\right)^2\right]$

$=\frac{1}{2}\left[3-\frac{1}{3}\right] = \frac{1}{2}\left(\frac{8}{3}\right) = \frac{4}{3}$

92) $\int \frac{2\cos x - e^{-ix}}{\cos x + i\sin x} dx$

$= \int \frac{2\cos x - (\cos x - i\sin x)}{\cos x + i\sin x} dx = \int \frac{2\cos x - \cos x + i\sin x}{\cos x + i\sin x} dx$

$= \int \frac{\cos x + i\sin x}{\cos x + i\sin x} dx = \int dx = x + c$

....

93) $\int \frac{\sqrt{\cot 2x}}{1 - \cos^2 2x} dx$

$= \int \frac{\sqrt{\cot 2x}}{\sin^2 2x} dx = \int \cot^{\frac{1}{2}} 2x \csc^2 2x dx$

$= -\frac{1}{2} \cdot \frac{2}{3} \cot^{\frac{3}{2}} 2x + c = -\frac{1}{3}\sqrt{\cot^3 2x} + c$

....

94) $\int \sqrt{\sec^2 x - 1} \tan x\, dx$

$= \int \sqrt{\tan^2 x} \tan x\, dx = \pm \int \tan^2 x\, dx$

$= \pm \int (\sec^2 x - 1) dx = \pm(\tan x - x) + c$

....

95) $\int \frac{1 - \cos x}{1 + \cos x} dx$

$= \int \frac{1 - \cos x}{1 + \cos x} \frac{1 - \cos x}{1 - \cos x} dx = \int \frac{1 - 2\cos x + \cos^2 x}{1 - \cos^2 x} dx$

$= \int \frac{1 - 2\cos x + \cos^2 x}{\sin^2 x} dx = \int (\csc^2 x - 2\csc x \cot x + \cot^2 x) dx$

$= \int (\csc^2 x - 2\csc x \cot x + \csc^2 x - 1) dx$

$= \int (2\csc^2 x - 2\csc x \cot x - 1) dx = -2\cot x + 2\csc x - x + c$

96) $\int sec^4 x dx$

$=\int sec^2 x sec^2 x\, dx = \int (1 + tan^2 x) sec^2 x dx$

$=\int (sec^2 x + tan^2 x sec^2 x) dx = tanx + \frac{1}{3} tan^3 x + c$

....

97) $\int tan^4 x dx$

$=\int tan^2 x tan^2 x dx = \int tan^2 x (sec^2 x - 1) dx$

$=\int (tan^2 x sec^2 x - tan^2 x) dx = \int (tan^2 x sec^2 x - sec^2 x + 1) dx$

$=\frac{1}{3} tan^3 x - tanx + x + c$

....

98) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-2sin^3 x}{sin^2 x} dx$

$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-2sinx(1-cos^2 x)}{sin^2 x} dx = \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1-2sinx+2cos^2 x\, sinx}{sin^2 x} dx$

$=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} (csc^2 x - 2cscx + 2cscx\, cotx) dx$

$=[-cotx + 2ln|cscx + cotx| - 2cscx]_{-\frac{\pi}{4}}^{\frac{\pi}{4}}$

$=\left[-1 + 2ln\left|\sqrt{2} + 1\right| - 2\sqrt{2}\right] - \left[1 + ln\left|-\sqrt{2} - 1\right| - 2\sqrt{2}\right]$

$=-1 + 2ln\left(\sqrt{2} + 1\right) - 2\sqrt{2} - 1 - 2ln\left(\sqrt{2} + 1\right) + 2\sqrt{2} = -2$

99) $\int \frac{1}{\sin 2x \cos 2x} dx$

$= \int \frac{\sin^2 2x + \cos^2 2x}{\sin 2x \cos 2x} dx = \int \left(\frac{\sin 2x}{\cos 2x} + \frac{\cos 2x}{\sin 2x}\right) dx$

$= -\frac{1}{2} \ln|\cos 2x| + \frac{1}{2} \ln|\sin 2x| + c$

....

100) $\int \frac{1}{\sec 2x \csc 2x} dx$

$= \int \sin 2x \cos 2x dx = \frac{1}{2} \int \sin 4x dx = -\frac{1}{8} \cos 4x + c$

OR: $\int \frac{1}{\sec 2x \csc 2x} dx = \int \sin 2x \cos 2x dx$

$= \frac{1}{4} \sin^2 2x + c$

....

101) $\int \frac{\cos 2x}{\cos^4 x} dx$

$= \int \frac{\cos^2 x - \sin^2 x}{\cos^4 x} dx = \int (sce^2 x - \tan^2 x \sec^2 x) dx$

$= \tan x - \frac{1}{3} \tan^3 x + c$

OR: $\int \frac{\cos 2x}{\cos^4 x} dx = \int \frac{2\cos^2 x - 1}{\cos^4 x} dx = \int (2\sec^2 x - \sec^4 x) dx$

$= \int \left(2\sec^2 x - \sec^2 x(\tan^2 x + 1)\right) dx$

$= \int (2\sec^2 x - \tan^2 x \sec^2 x - \sec^2 x) dx$

$= \int (\sec^2 x - \tan^2 x \sec^2 x) dx = \tan x - \frac{1}{3} \tan^3 x + c$

102) $\int(\sec 2x \csc 2x)dx$

$$=\int \frac{1}{\sin 2x\cos 2x}dx=\int \frac{2}{2\sin 2x\cos 2x}dx=2\int \frac{1}{\sin 4x}dx=2\int \csc 4x dx$$

$$=-\frac{1}{2}\ln|\csc 4x+\cot 4x|+c=\ln\left|\frac{1}{\sqrt{\csc 4x+\cot 4x}}\right|+c$$

---

103) $\int(\sec 2x+\csc 2x)dx$

$$=\frac{1}{2}\ln|\sec 2x+\tan 2x|-\frac{1}{2}\ln|\csc 2x+\cot 2x|+c$$

$$=\ln\left|\sqrt{\sec 2x+\tan 2x}\right|-\ln\left|\sqrt{\csc 2x+\cot 2x}\right|+c$$

$$=\ln\left|\sqrt{\frac{\sec 2x+\tan 2x}{\csc 2x+\cot 2x}}\right|+c$$

---

104) $\int(\sec 2x-\csc 2x)dx$

$$=\frac{1}{2}\ln|\sec 2x+\tan 2x|+\frac{1}{2}\ln|\csc 2x+\cot 2x|+c$$

$$=\ln\left|\sqrt{\sec 2x+\tan 2x}\right|+\ln\left|\sqrt{\csc 2x+\cot 2x}\right|+c$$

$$=\ln\left|\sqrt{(\sec 2x+\tan 2x)(\csc 2x+\cot 2x)}\right|+c$$

---

105) $\int \frac{\sec 2x}{\csc 2x}dx$

$$=\int \frac{\frac{1}{\cos 2x}}{\frac{1}{\sin 2x}}dx=\int \frac{\sin 2x}{\cos 2x}dx=-\frac{1}{2}\ln|\cos 2x|+c$$

106) $\int \frac{cssc2x}{sec2x}dx$

$=\int \frac{\frac{1}{sin2x}}{\frac{1}{cos2x}}dx = \int \frac{cos2x}{sin2x}dx = \frac{1}{2}ln|sin2x| + c$

....

107) $\int(csc2x - sec2x)dx$

$=-\frac{1}{2}ln|csc2x + cot2x| - \frac{1}{2}ln|sec2x + tan2x| + c$

....

108) $\int \frac{sin^3x}{cosx - cos^2x}dx$

$=\int \frac{(1-cos^2x)sinx}{cosx(1-cosx)}dx = \int \frac{(1-cosx)(1+cosx)sinx}{cosx(1-cosx)}dx$

$=\int \frac{(1+cosx)sinx}{cosx}dx = \int \frac{sinx}{cosx}dx + \int sinxdx$

$= -ln|cosx| - cosx + c$

....

109) $\int \sqrt{sinx}\, cos^3xdx$

$=\int (sinx)^{\frac{1}{2}}(1 - sin^2x)cosxdx$

$= \int \left(sin^{\frac{1}{2}}x\, cosx - sin^{\frac{5}{2}}x\, cosx\right)dx$

$= \frac{2}{3}sin^{\frac{3}{2}}x - \frac{2}{7}sin^{\frac{7}{2}}x + c = \frac{2}{3}\sqrt{sin^3x} - \frac{2}{7}\sqrt{sin^7x} + c$

110) $\int_0^{\frac{\pi}{16}} sec^2 4x\, dx$

$=\left[\frac{1}{4}tan4x\right]_0^{\frac{\pi}{16}} = \frac{1}{4}\left(tan\frac{\pi}{4} - tan0\right) = \frac{1}{4}(1) = \frac{1}{4}$

....

111) $\int sin^4 x cos^4 x\, dx$

$=\frac{1}{16}\int 16sin^4 xcos^4 xdx = \frac{1}{16}\int sin^4 2xdx$

$=\frac{1}{16}\int\left(\frac{1}{2} - \frac{1}{2}cos4x\right)^2 dx = \frac{1}{16}\int\left(\frac{1}{4} - \frac{1}{2}cos4x + \frac{1}{4}cos^2 4x\right)dx$

$=\frac{1}{16}\int\left(\frac{1}{4} - \frac{1}{2}cos4x + \frac{1}{8} + \frac{1}{8}cos8x\right)dx$

$=\frac{1}{16}\left(\frac{3}{8}x - \frac{1}{8}sin4x + \frac{1}{64}sin8x\right) + c$

....

112) $\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} \frac{dx}{sin^2 2x}$

$=\int_{\frac{\pi}{8}}^{\frac{\pi}{4}} csc^2 2x\, dx = \left[-\frac{1}{2}cot2x\right]_{\frac{\pi}{8}}^{\frac{\pi}{4}}$

$=-\frac{1}{2}\left[cot\frac{\pi}{2} - cot\frac{\pi}{4}\right] = -\frac{1}{2}(0-1) = \frac{1}{2}$

....

113) $\int \frac{1}{1+cot^2 x}dx$

$=\int \frac{1}{csc^2 x}dx = \int sin^2 xdx = \frac{1}{2}\int(1-cos2x)dx$

$=\frac{1}{2}\left(x - \frac{1}{2}sin2x\right) + c = \frac{1}{2}x - \frac{1}{4}sin2x + c$

114) $\int_0^{\frac{\pi}{2}} sin^2 x dx$

$=\int_0^{\frac{\pi}{2}} \frac{1}{2}(1-cos2x)dx = \left[\frac{1}{2}x - \frac{1}{4}sin2x\right]_0^{\frac{\pi}{2}}$

$=\left(\frac{\pi}{4} - \frac{1}{4}sin\pi\right) - \left(0 - \frac{1}{4}sin0\right) = \frac{\pi}{4}$

---

115) $\int \frac{cos4x}{cosx} dx$

$=\int \frac{1-2sin^2 2x}{cosx} dx = \int \frac{1-2(2sinx\, cosx)^2}{cosx} dx$

$=\int \frac{1-8sin^2 x cos^2 x}{cosx} dx = \int (secx - 8sin^2 x\, cosx)dx$

$=ln|secx + tanx| - \frac{8}{3}sin^3 x + c$

116) $\int \frac{2sin\sqrt[3]{x}}{\sqrt[3]{x^2}} dx$

$=2(3)\int sin\sqrt[3]{x}.\frac{1}{3\sqrt[3]{x^2}} dx = -6cos\sqrt[3]{x} + c$

117) $\int tan^7 x dx$

$=\int tan^5 x tan^2 x dx = \int tan^5 x(sec^2 x - 1)dx$

$=\int \left(tan^5 x sec^2 x - tan^5 x\right)dx$

$= \int (tan^5 x sec^2 x - tan^3 x(sec^2 x - 1))$

$= \int (tan^5 x sec^2 x - tan^3 x sec^2 x + tan^3 x)$

$= \int (tan^5 x sec^2 x - tan^3 x sec^2 x + tanx sec^2 x - tanx)dx$

$=\frac{1}{6}tan^6 x - \frac{1}{4}tan^4 x + \frac{1}{2}tan^2 x + ln|cosx| + c$

118) $\int_0^{\pi} \sqrt{1-sin2x}dx$

$=\int_0^{\pi} \sqrt{cos^2x - 2sinx\,cosx + sin^2x}dx = \int_0^{\pi} \sqrt{(cosx - sinx)^2}dx$

$=\int_0^{\pi} |cosx - sinx|dx \quad , cosx - sinx = 0 \rightarrow tanx = 1 \rightarrow x = \frac{\pi}{4} \in [0, \pi]$

$=\int_0^{\pi} |cosx - sinx|dx = \int_0^{\frac{\pi}{4}} (cosx - sinx)dx + \int_{\frac{\pi}{4}}^{\pi} (sinx - cosx)dx$

$=[sinx + cosx]_0^{\frac{\pi}{4}} + [-cosx - sinx]_{\frac{\pi}{4}}^{\pi}$

$=\left[\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) - (0+1)\right] + \left[(-(-1) - 0) - \left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)\right]$

$=(\sqrt{2} - 1) + (1 + \sqrt{2}) = 2\sqrt{2}$

....

119) $\int tan^3 x dx$

$=\int tan^2x\, tanxdx = \int tanx(sec^2x - 1)dx$

$=\int (tanxsec^2x - tanx)\, dx = \frac{1}{2}tan^2x + ln|cosx| + c$

....

120) $\int cotx\, sec^2 x dx$

$=\int \frac{cosx}{sinx} \cdot \frac{1}{cos^2x} dx = \int \frac{1}{sinxcosx} dx = \int \frac{2}{2sinxcosx} dx$

$=2\int \frac{1}{sin2x} dx = 2\int csc2xdx = -ln|csc2x + cot2x| + c$

....

121) $\int tan^{-3}xsec^2xdx = -\frac{1}{2}tan^{-2}x + c = -\frac{1}{2tan^2x} + c$

122) $\int cosy\, cos5ydy$

$=\frac{1}{2}\int(cos5y+cos3y)dy=\frac{1}{2}\left(\frac{1}{5}sin5y+\frac{1}{3}sin3y\right)+c$

$=\frac{1}{10}sin5y+\frac{1}{6}sin3y+c$

....

123) $\int sin4y\, cos5y\, dy$

$=\int\frac{1}{2}(sin9y+siny)dy=\frac{1}{2}\left(-\frac{1}{9}cos9y-cosy\right)+c$

$=-\frac{1}{18}cos9y-\frac{1}{2}cosy+c$

....

124) $\int sec^4\theta d\theta$

$=\int sec^2\theta sec^2\theta d\theta=\int sec^2\theta(tan^2\theta+1)d\theta$

$=\int(tan^2\theta sec^2\theta+sec^2\theta)d\theta=\frac{1}{3}tan^3\theta+tan\theta+c$

....

125) $\int tan^3 3x sec^3 3xdx$

$=\int tan^2 3x\, tan3xsec^3 3xdx=\int(sec^2 3x-1)tan3xsec^3 3xdx$

$=\int(sec^5 3x\, tan3x-sec^3 3x\, tan3x)dx$

$=\int(sec^4 3x\, sec3x\, tan3x-sec^2 3x\, sec3x\, tan3x)dx$

$=\frac{1}{15}sec^5 3x-\frac{1}{9}sec^3 3x+c$

126) $\int \frac{cot(7x+5)sin(3x+2)}{csc(7x+5)}dx$

$=\int \frac{cos(7x+5)}{sin(7x+5)}.sin(7x+5)sin(3x+2)dx$

$=\int cos(7x+5).sin(3x+2)dx$

$=\frac{1}{2}\int[sin(10x+7)+sin(4x+3)]dx$

$=\frac{1}{2}\left[-\frac{1}{10}cos(10x+7)-\frac{1}{4}cos(4x+3)\right]+c$

$=-\frac{1}{20}cos(10x+7)-\frac{1}{8}cos(4x+3)+c$

....

127) $\int(sin2x+cos2x)^2dx$

$=\int(sin^2 2x+2sin2xcos2x+cos^2 2x)dx$

$=\int(1+sin4x)dx=x-\frac{1}{4}cos4x+c$

....

128) $\int(tanx+secx)^2dx$

$=\int(tan^2x+2tanxsecx+sec^2x)dx$

$=\int(sec^2x-1+2tanxsecx+sec^2x)dx$

$=\int(2sec^2x-1+2tanxsecx)dx=2tanx+2secx-x+c$

....

129) $\int sinx\, cos\frac{x}{2}dx$

$=\int 2sin\frac{x}{2}\, cos\frac{x}{2}\, cos\frac{x}{2}dx=2\int cos^2\frac{x}{2}sin\frac{x}{2}dx$

$=-4.\frac{1}{3}cos^3\frac{x}{2}+c=-\frac{4}{3}cos^3\frac{x}{2}+c$

130) $\int \frac{2sin2x}{\sqrt{cosx}}dx$

$=\int \frac{4sinx\,cosx}{\sqrt{cosx}}dx = 4\int \frac{sinx\sqrt{cosx}\sqrt{cosx}}{\sqrt{cosx}}dx = 4\int \sqrt{cosx}\,sinx\,dx$

$=4\int (cosx)^{\frac{1}{2}}sinx\,dx = 4.\left(-\frac{2}{3}\right)(cosx)^{\frac{3}{2}} + c = -\frac{8}{3}\sqrt{cos^3x} + c$

---

131) $\int cos^2\frac{x}{3}dx$

$=\int \frac{1}{2}\left(1 + cos\frac{2x}{3}\right)dx = \frac{1}{2}\left(x + \frac{3}{2}sin\frac{2x}{3}\right) + c$

$=\frac{x}{2} + \frac{3}{4}\,sin\frac{2x}{3} + c$

132) $\int cos^3 2x\,sinxdx$

$=\int (cos2x)^3 sinxdx = \int (2cos^2x - 1)^3 sinxdx$

$=\int (4cos^4x - 4cos^2x + 1)(2cos^2x - 1)sinxdx$

$=\int (8cos^6x - 8cos^4x + 2cos^2x - 4cos^4x + 4cos^2x - 1)sinxdx$

$=\int (8cos^6x - 12cos^4x + 6cos^2x - 1)sinxdx$

$=\int (8cos^6xsinx - 12cos^4xsinx + 6cos^2xsinx - sinx)dx$

$=-\frac{8}{7}cos^7x + \frac{12}{5}\,cos^5x - 2cos^3x + cosx + c$

133) $\int \frac{sec^4x}{\sqrt{tanx}}dx$

$=\int tan^{\frac{-1}{2}}x\,sec^2xsec^2xdx = \int tan^{\frac{-1}{2}}x\,sec^2x(tan^2x + 1)dx$

$=\int \left(tan^{\frac{3}{2}}x\,sec^2x + tan^{\frac{-1}{2}}x\,sec^2x\right)dx = \frac{2}{5}tan^{\frac{5}{2}}x + 2tan^{\frac{1}{2}}x + c$

$=\frac{2}{5}\sqrt{tan^5x} + 2\sqrt{tanx} + c$

134) $\int \frac{\cos^3 x}{\sqrt{\sin x}}dx$

$=\int \sin^{\frac{-1}{2}}x\, \cos x \cos^2 x dx = \int \sin^{\frac{-1}{2}}x\, \cos x(1-\sin^2 x)dx$

$=\int \left(\sin^{\frac{-1}{2}}x\, \cos x - \sin^{\frac{3}{2}}x\, \cos x\right)dx = 2\sin^{\frac{1}{2}}x - \frac{2}{5}\sin^{\frac{5}{2}}x + c$

$=2\sqrt{\sin x} - \frac{2}{5}\sqrt{\sin^5 x} + c$

135) $\int \sin^2 x \cos^4 x dx$

$=\int \left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)\left(\frac{1}{2}+\frac{1}{2}\cos 2x\right)^2 dx$

$=\int \left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)\left(\frac{1}{4}+\frac{1}{2}\cos 2x+\frac{1}{4}\cos^2 2x\right)dx$

$=\int \left(\frac{1}{8}+\frac{1}{4}\cos 2x+\frac{1}{8}\cos^2 2x-\frac{1}{8}\cos 2x-\frac{1}{4}\cos^2 2x-\frac{1}{8}\cos^3 2x\right)dx$

$=\int \left(\frac{1}{8}+\frac{1}{8}\cos 2x-\frac{1}{8}\cos^2 2x-\frac{1}{8}\cos^3 2x\right)dx$

$=\int \left(\frac{1}{8}+\frac{1}{8}\cos 2x-\frac{1}{8}\left(\frac{1}{2}+\frac{1}{2}\cos 4x\right)-\frac{1}{8}(1-\sin^2 2x)\cos 2x\right)dx$

$=\int \left(\frac{1}{8}+\frac{1}{8}\cos 2x-\frac{1}{16}-\frac{1}{16}\cos 4x-\frac{1}{8}\cos 2x+\frac{1}{8}\sin^2 2x\, \cos 2x\right)dx$

$=\int \left(\frac{1}{16}-\frac{1}{16}\cos 4x+\frac{1}{8}\sin^2 2x\, \cos 2x\right)dx$

$=\frac{x}{16}-\frac{1}{64}\sin 4x+\frac{1}{48}\sin^3 2x+c$

....

136) $\int \sin^3 x \cos^2 x dx$

$=\int \cos^2 x\, \sin x(1-\cos^2 x)dx = \int(\cos^2 x\, \sin x - \cos^4 x\, \sin x)dx$

$=-\frac{1}{3}\cos^3 x+\frac{1}{5}\cos^5 x+c$

137) $\int sec^3x tan^3x dx$

$=\int sec^3x\, tanx tan^2x dx = \int sec^3x\, tanx(sec^2x - 1)dx$

$=\int \left(sec^5x\, tanx - sec^3x\, tanx\right)$

$=\int (sec^4x\, secx\, tanx - sec^2x secx\, tanx)dx$

$=\frac{1}{5}sec^5x - \frac{1}{3}sec^3x + c$

....

138) $\int tan^5x sec^{\frac{-3}{2}}x dx$

$=\int tan^3x(sec^2x - 1)sec^{\frac{-3}{2}}x dx$

$=\int \left(tan^3x sec^{\frac{1}{2}}x - tan^3x sec^{\frac{-3}{2}}x\right)dx$

$=\int \left[(sec^2x - 1)tanx sec^{\frac{1}{2}}x - (sec^2x - 1)tanx sec^{\frac{-3}{2}}x\right]dx$

$=\int \left[sec^{\frac{3}{2}}x secx\, tanx - 2sec^{\frac{-1}{2}}x\, secx\, tanx + sec^{\frac{-5}{2}}x secx\, tanx\right]dx$

$=\frac{2}{5}sec^{\frac{5}{2}}x - 4sec^{\frac{1}{2}}x - \frac{2}{3}sec^{\frac{-3}{2}}x + c$

$=\frac{2}{5}\sqrt{sec^5x} - 4\sqrt{secx} - \frac{2}{3}\frac{1}{\sqrt{sec^3x}} + c$

....

139) $\int sinx\, cosx dx$

$=\frac{1}{2}sin^2x + c$, **OR**: $\frac{-1}{2}cos^2x + c$

140) $\int_0^{\frac{\pi}{2}} cos^2x\, cos2x dx$

$=\int_0^{\frac{\pi}{2}} cos^2x(1-2sin^2x)dx$

$=\int_0^{\frac{\pi}{2}}(cos^2x-2sin^2xcos^2x\,)dx$

$=\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}+\frac{1}{2}cos2x-\frac{1}{2}(4sin^2xcos^2x)\right)dx$

$=\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}+\frac{1}{2}cos2x-\frac{1}{2}(sin^22x)\right)dx$

$=\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}+\frac{1}{2}cos2x-\frac{1}{2}(\frac{1}{2}-\frac{1}{2}cos4x)\right)dx$

$=\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}+\frac{1}{2}cos2x-\frac{1}{4}+\frac{1}{4}cos4x)\right)dx$

$=\int_0^{\frac{\pi}{2}}\left(\frac{1}{4}+\frac{1}{2}cos2x+\frac{1}{4}cos4x)\right)dx$

$=\left[\frac{x}{4}+\frac{1}{4}sin2x+\frac{1}{16}sin4x\right]_0^{\frac{\pi}{2}}=\left(\frac{\pi}{8}+0+0\right)-(0)=\frac{\pi}{8}$

---

141) $\int sin2xcosxdx$

$=\int 2sinx\, cosx\, cosx\, dx$

$=2\int cos^2x\, sinx\, dx$

$=-\frac{2}{3}cos^3x+c$

142) $\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \sin^2 x \cos 2x dx$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \sin^2 x(2\cos^2 x - 1)dx$$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} (2\sin^2 x\cos^2 x - \sin^2 x)dx$$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{2}(4\sin^2 x\cos^2 x) - (\frac{1}{2} - \frac{1}{2}\cos 2x)\right) dx$$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{2}\ \sin^2 2x - \frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{2}\ (\frac{1}{2} - \frac{1}{2}\cos 4x) - \frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{1}{4} - \frac{1}{4}\cos 4x - \frac{1}{2} + \frac{1}{2}\cos 2x\right) dx$$

$$=\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \left(\frac{-1}{4} - \frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x\right) dx$$

$$=\left[-\frac{x}{4} - \frac{1}{16}\sin 4x + \frac{1}{4}\sin 2x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}}$$

$$=\left[-\frac{3\pi}{4} - 0 - \frac{1}{4}\right] - \left[0 - 0 - \frac{\pi}{2}\right] = \frac{-4-3\pi+8\pi}{16} = \frac{5\pi-4}{16}$$

143) $\int \sqrt{tanx} sec^4x\, dx$

$= \int tan^{\frac{1}{2}}x sec^2x sec^2x\, dx = \int tan^{\frac{1}{2}}x sec^2x(tan^2x+1)dx$

$= \int \left(tan^{\frac{5}{2}}x sec^2x + tan^{\frac{1}{2}}x sec^2x\right)dx$

$= \frac{2}{7} tan^{\frac{7}{2}}x + \frac{2}{3} tan^{\frac{3}{2}}x + c = \frac{2}{7}\sqrt{tan^7x} + \frac{2}{3}\sqrt{tan^3x} + c$

144) $\int cos2x\, sinx dx$

$= \int (2cos^2x - 1)sinx dx = \int (2cos^2x sinx - sinx)dx$

$= -\frac{2}{3}\, cos^3x + cosx + c$

145) $\int cos2x sin2x dx$

$= \frac{1}{4} sin^2 2x + c$, **OR**: $-\frac{1}{4}\, cos^2 2x + c$

146) $\int sin^3x cos^2x dx$

$= \int sin^2x\, sinx cos^2x dx$

$= \int (1 - cos^2x)sinx cos^2x dx$

$= \int (cos^2x sinx - cos^4x sinx)dx$

$= -\frac{1}{3}\, cos^3x + \frac{1}{5}\, cos^5x + c$

147) $\int \frac{cosx}{sin^4x}dx$

$=\int sin^{-4}x\ cosxdx = -\frac{1}{3}sin^{-3}x + c$

$=\frac{-1}{3sin^3x} + c = -\frac{1}{3}csc^3x + c$

OR: $\int \frac{cosx}{sin^4x}dx = \int csc^3x\ cotx\ dx = \int csc^2x\ cscxcotxdx$

$=-\frac{1}{3}\ csc^3x + c$

....

148) $\int \frac{sin^3x-4}{1-cos^2x}dx$

$=\int \frac{sin^3x}{1-cos^2x}dx - \int \frac{4}{1-cos^2x}dx$

$=\int \frac{sin^3x}{sin^2x}dx - \int \frac{4}{sin^2x}dx = \int sinxdx - 4\int csc^2xdx$

$=-cosx + 4cotx + c$

....

149) $\int cos^4x\ tanxdx$

$=\int cos^4x.\frac{sinx}{cosx}dx = \int cos^3xsinxdx = -\frac{1}{4}cos^4x + c$

....

150) $\int sec^4xtanxdx$

$=\int sec^3x\ secx\ tanx\ dx = \frac{1}{4}sec^4x + c$

151) $\int \frac{\sin 3x}{\sqrt[3]{\cos^4 3x}} dx$

$= \int (\cos 3x)^{\frac{-4}{3}} \sin 3x dx = -\frac{1}{3}.(-3)(\cos 3x)^{\frac{-1}{3}} + c$

$= \frac{1}{(\cos 3x)^{\frac{1}{3}}} + c = \frac{1}{\sqrt[3]{\cos 3x}} + c$

---

152) $\int \frac{\tan^3 x}{\cos^2 x} dx$

$= \int \tan^3 x \sec^2 x dx = \frac{1}{4} \tan^4 x + c$

---

153) $\int \frac{\cos^3 x}{\sin^4 x} dx$

$= \int \cot^3 x \csc x dx = \int \cot^2 x \cot x \csc x \, dx$

$= \int (\csc^2 x - 1) \cot x \csc x dx = \int (\csc^2 x \csc x \cot x - \csc x \cot x) dx$

$= -\frac{1}{3} \csc^3 x + \csc x + c$

---

154) $\int \frac{\sin x}{1 - \sin^2 x} dx$

$= \int \frac{\sin x}{\cos^2 x} dx = \int \sec x \tan x dx = \sec x + c$

155) $\int sin2xsin^2xdx$

$=\int 2sinx\, cosxsin^2xdx = 2\int sin^3xcosxdx = \frac{1}{2}sin^4x + c$

....

156) $\int \frac{secx}{cotx}dx$

$=\int secx\, tanx\, dx = secx + c$

....

157) $\int \frac{cos2x+2sin^2x}{cos^2x}dx$

$=\int \frac{cos^2x - sin^2x+2sin^2x}{cos^2x}dx = \int \frac{cos^2x+sin^2x}{cos^2x}dx$

$=\int \frac{1}{cos^2x}dx = \int sec^2x\, dx = tanx + c$

....

158) $\int \frac{sin2x}{sinx}dx$

$=\int \frac{2sinxcosx}{sinx}dx = 2\int cosx\, dx = 2sinx + c$

....

159) $\int sin6xcos^23xdx$

$=\int 2sin3x\, cos3xcos^23xdx = 2\int cos^33xsin3xdx$

$=-\frac{2}{3}.\frac{1}{4}\, cos^43x + c$

$=-\frac{1}{6}\, cos^43x + c$

160) $\int \frac{\cos^3 x+\sin^3 x}{\cos x+\sin x}dx$

$=\int \frac{(\cos x+\sin x)(\cos^2 x-\cos x\sin x+\sin^2 x)}{\cos x+\sin x}dx$

$=(\cos^2 x-\cos x\sin x+\sin^2 x)dx=\int(1-\sin x\cos x)dx$

$=x-\frac{1}{2}\sin^2 x+c$

....

161) $\int \sin 2x(\cos x+\sin x)dx$

$=\int 2\sin x\cos x(\sin x+\cos x)dx$

$=2\int(\sin^2 x\cos x+\cos^2 x\sin x)dx$

$=2\left[\frac{1}{3}\ \sin^3 x-\frac{1}{3}\cos^3 x\right]+c=\frac{2}{3}\sin^3 x-\frac{2}{3}\cos^3 x+c$

....

162) $\int \frac{\sin^3 x}{\cos^4 x}dx$

$=\int \tan^3 x\sec x dx=\int \tan^2 x\sec x\tan x dx$

$=\int(\sec^2 x-1)\sec x\tan x dx$

$=\int(\sec^2 x\sec x\tan x-\sec x\tan x)dx=\frac{1}{3}\sec^3 x-\sec x+c$

163) $\int \frac{\cos x+\sin x}{\sqrt[4]{\cos x-\sin x}}dx$

$=\int(\cos x-\sin x)^{\frac{-1}{4}}(\cos x+\sin x)dx$

$=-\frac{4}{3}(\cos x-\sin x)^{\frac{3}{4}}+c=-\frac{4}{3}\sqrt[4]{(\cos x-\sin x)^3}+c$

164) $\int \frac{cos^2x}{1+sinx}dx$

$=\int \frac{1-sin^2x}{1+sinx}dx = \int \frac{(1-sinx)(1+sinx)}{1+sinx}dx = \int(1-sinx)dx$

$=x + cosx + c$

OR: $\int \frac{cos^2x}{1+sinx}dx = \int \frac{cos^2x}{1+sinx}.\frac{1-sinx}{1-sinx}dx$

$=\int \frac{cos^2x(1-sinx)}{1-sin^2x}dx = \int \frac{cos^2x(1-sinx)}{cos^2x}dx$

$=\int(1-sinx)dx = x + cosx + c$

....

165) $\int\left(sin^2 2x - cosx\right)dx$

$=\int\left(\frac{1}{2}-\frac{1}{2}cos4x - cos2x\right)dx$

$=\frac{x}{2}-\frac{1}{8}sin4x-\frac{1}{2}sin2x + c$

....

166) $\int \frac{sin3x}{sec^2 3x}dx$

$=\int cos^2 3x\, sin3xdx = -\frac{1}{9}cos^3 3x + c$

....

167) $\int(1+2sin4x)^2cos4xdx$

$=\frac{1}{24}\,(1+2sin4x)^3 + c$

168) $\int \frac{sin2x}{\sqrt{1+2sin^2x}}dx$

$=\int (1+2sin^2x)^{\frac{-1}{2}} sin2xdx = \int (1+2sin^2x)^{\frac{-1}{2}} 2sinxcosxdx$

$=\sqrt{1+2sin^2x}+c$

....

169) $\int \sqrt{1+cos2x}\, sin2xdx$

$=\int (1+cos2x)^{\frac{1}{2}} sin2xdx = -\frac{1}{2}.\frac{2}{3}(1+cos2x)^{\frac{3}{2}}+c$

$=-\frac{1}{3}\sqrt{cos^3 2x}+c$

....

170) $\int \sqrt{1-cos2x}\, cosxdx$

$=\int \sqrt{1-1+2sin^2x} cosxdx = \int \sqrt{2sin^2x}. cosxdx$

$=\pm\sqrt{2}\ \int sinxcosxdx = \pm\frac{1}{\sqrt{2}}sin^2x+c$

....

171) $\int \frac{csc^2 3x}{\sqrt{1+2cot3x}}dx$

$=\int (1+2cot3x)^{\frac{-1}{2}} csc^2 3xdx$

$=-\frac{1}{6}.2(1+2cot3x)^{\frac{1}{2}}+c = -\frac{1}{3}\sqrt{1+2cot3x}+c$

172) $\int \frac{\sin^2 2x}{1+\cos 2x} dx$

$=\int \frac{1-\cos^2 2x}{1+\cos 2x} dx = \int \frac{(1-\cos 2x)(1+\cos 2x)}{1+\cos 2x} dx$

$=\int (1-\cos 2x) = x - \frac{1}{2}\sin 2x + c$

....

173) $\int \frac{\sqrt{\tan x+1}}{\cos^2 x} dx$

$=\int (\tan x+1)^{\frac{1}{2}} \sec^2 x dx = \frac{2}{3}(\tan x+1)^{\frac{3}{2}} + c = \frac{2}{3}\sqrt{(\tan x+1)^3} + c$

....

174) $\int (1+\cos x)^2 dx$

$=\int (1+2\cos x+\cos^2 x) dx = \int \left(1+2\cos x+\frac{1}{2}+\frac{1}{2}\cos 2x\right) dx$

$=\frac{3}{2}x + 2\sin x + \frac{1}{4}\sin 2x + c$

....

175) $\int (\cos x - \sin 2x)^2 dx$

$=\int (\cos^2 x - 2\cos x \sin 2x + \sin^2 2x) dx$

$=\int \left(\frac{1}{2}+\frac{1}{2}\cos 2x - 4\cos^2 x \sin x + \frac{1}{2} - \frac{1}{2}\cos 4x\right) dx$

$=\int \left(1+\frac{1}{2}\cos 2x - 4\cos^2 x \sin x - \frac{1}{2}\cos 4x\right) dx$

$=x + \frac{1}{4}\sin 2x + \frac{4}{3}\cos^3 x - \frac{1}{8}\sin 4x + c$

176) $\int \frac{\cos^4x-\sin^4x}{\sin^4 2x}dx$

$=\int \frac{\cos^4x-\sin^4x}{16\sin^4x\cos^4x}dx = \frac{1}{16}\int\left(\frac{1}{\sin^4x}-\frac{1}{\cos^4x}\right)dx$

$=\frac{1}{16}\int(\csc^4x-\sec^4x)dx$

$= \frac{1}{16}\int\Big(\csc^2x(\cot^2x+1)-\sec^2x(\tan^2x+1)\Big)dx$

$=\frac{1}{16}\int(\cot^2x\csc^2x+\csc^2x-\tan^2x\sec^2x-\sec^2x)dx$

$=\frac{1}{16}\left[-\frac{1}{3}\cot^3x-\cot x-\frac{1}{3}\tan^3x-\tan x\right]+c$

177) $\int \frac{\sin 2x}{\cos^2 2+2\cos 2x+1}dx$

$=\int \frac{\sin 2x}{(\cos 2x+1)^2}dx = \int(\cos 2x+1)^{-2}\sin 2x\,dx$

$=\frac{-1}{2}\;\frac{(\cos 2x+1)^{-1}}{-1}+c = \frac{1}{2(\cos 2x+1)}+c$

178) $\int \cos^3x\sin^2x\,dx$

$=\int \cos^2x\cos x\sin^2x\,dx = \int(1-\sin^2x)\sin^2x\cos x\,dx$

$=\int(\sin^2x\cos x-\sin^4x\cos x)dx$

$=\frac{1}{3}\sin^3x-\frac{1}{5}\sin^5x+c$

179) $\int \frac{tanx}{1+cos2x} dx$

$= \int \frac{tanx}{1+2cos^2x-1} dx = \int \frac{tanx}{2cos^2x} dx = \frac{1}{2} \int tanx sec^2x \, dx$

$= \frac{1}{4} tan^2x + c$

....

180) $\int sin4x \, cos2x dx$

$= \int 2sin2x \, cos2x \, cos2x dx = \int cos^2 2x \, sin2x \, 2dx$

$= -\frac{1}{3} cos^3 2x + c$

....

181) $\int cos4x \sqrt{2 - sin4x} \, dx$

$= \int (2 - sin4x)^{\frac{1}{2}} cos4x \, dx = -\frac{1}{4} . \frac{2}{3} (2 - sin4x)^{\frac{3}{2}} + c$

$= -\frac{1}{6} \sqrt{(2 - sin4x)^3 + c}$

....

182) $\int sinxcosx(sinx + cosx)dx$

$= \int (sin^2xcosx + cos^2xsinx)dx$

$= \frac{1}{3} sin^3x - \frac{1}{3} cos^3x + c$

183) $\int \frac{dx}{cosxcotx\sqrt{secx+1}}$

$= \int \frac{1}{\sqrt{secx+1}} . \frac{1}{cosxcotx} dx = \int (secx + 1)^{\frac{-1}{2}} secx \, tanxdx$

$= 2(secx + 1)^{\frac{1}{2}} + c = 2\sqrt{secx + 1} + c$

184) $\int \frac{dx}{\sin^2 x-\cos^2 x}$

$=\int -\frac{dx}{\cos^2 x-\sin^2 x}dx = -\int \frac{dx}{\cos 2x} = -\int \sec 2x dx$

$= -\frac{1}{2}\ln|\sec 2x + \tan 2x| + c$

....

185) $\int \sin x\cos^3 x\tan^2 x\csc^4 x dx$

$=\int \sin x\cos^3 . \frac{\sin^2 x}{\cos^2 x}.\frac{1}{\sin^4 x}dx = \int \frac{\cos x}{\sin x}dx = \ln|\sin x| + c$

....

186) $\int \frac{\sin 3x}{5-2\cos 3x}dx$

$=\frac{1}{6}\int \frac{6\sin 3x}{5-2\cos 3x}dx = \frac{1}{6}\ln|5-2\cos 3x| + c$

....

187) $\int \frac{\sec^2 3x}{\tan 3x}dx$

$=\frac{1}{3} \int \frac{3\sec^2 3x}{\tan 3x}dx = \frac{1}{3}\ln|\tan 3x| + c$

....

188) $\int e^{\tan x^2}(x\sec^2 x^2)dx$

$=\frac{1}{2} e^{\tan x^2} + c$

....

189) $\int \cot x dx$

$=\int \frac{\cos x}{\sin x}dx = \ln|\sin x| + c$

190) $\int \frac{cos^3x-1}{cos^2x}dx$

$=\int \left(\frac{cos^3x}{cos^2x}-\frac{1}{cos^2x}\right)dx=\int(cosx-sec^2x)dx=sinx-tanx+c$

....

191) $\int cos6x\ sin3x\ dx$

$=\int(2cos^2 3x-1)sin3xdx=\int(2cos^2 3x\ sin3x-sin3x)dx$

$=-\frac{2}{3}.\frac{1}{3}cos^3 3x+\frac{1}{3}cos3x+c=-\frac{2}{9}cos^3 3x+\frac{1}{3}cos3x+c$

....

192) $\int cos3x\ cos\frac{3x}{2}\ dx$

$=\int\left(1-2sin^2\frac{3x}{2}\right)cos\frac{3x}{2}dx=\int\left(cos\frac{3x}{2}-2\ sin^2\frac{3x}{2}\ cos\frac{3x}{2}\right)dx$

$=\frac{2}{3}sin\frac{3x}{2}-\frac{4}{9}sin^3\frac{3x}{2}+c$

....

193) $\int cos^2\frac{x}{3}sin^2\frac{x}{3}dx$

$=\frac{1}{4}\int cos^2\frac{x}{3}sin^2\frac{x}{3}dx=\frac{1}{4}\int sin^2\frac{2x}{3}dx=\frac{1}{4}\int\left(\frac{1}{2}-\frac{1}{2}cos\frac{4x}{3}\right)dx$

$=\frac{1}{4}\left(\frac{x}{2}-\frac{1}{2}.\frac{3}{4}sin\frac{4x}{3}\right)+c=\frac{x}{8}-\frac{3}{64}sin\frac{4x}{3}+c$

....

194) $\int\sqrt{2-2cos2x}\ dx$

$=\int\sqrt{2-2(1-2sin^2x)}dx=\int\sqrt{2-2+4sin^2x}\ \ dx$

$=\int\sqrt{4sin^2x}\ dx=\pm2\int sinxdx=\pm2cosx+c$

195) $\int_0^{\frac{\pi}{2}} e^{cosx} sinx dx$

$= -[e^{cosx}]_0^{\frac{\pi}{2}} = -\left[e^{cos\frac{\pi}{2}} - e^{cos0}\right] = -(e^0 - e^1) = e - 1$

196) $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{sec^2x}{2+tanx} dx$

$= [ln|2 + tanx|]_{-\frac{\pi}{4}}^{\frac{\pi}{4}} = \left(ln(2 + tan\frac{\pi}{4})\right) - \left(ln(2 - tan\frac{\pi}{4})\right)$

$= ln(2 + 1) - ln(2 - 1) = ln3 - ln1 = ln3$

....

197) $\int sec^2 3x csc^2 3x\, dx$

$= \int (tan^2 3x + 1) csc^2 3x\, dx = \int (tan^2 3x csc^2 3x + csc^2 3x) dx$

$= \int \left(\frac{sin^2 3x}{cos^2 3x} \frac{1}{sin^2 3x} + csc^2 3x\right) dx = \int (sec^2 3x + csc^2 3x) dx$

$= \frac{1}{3} tan3x - \frac{1}{3} cot3x + c$

OR: $\int sec^2 3x csc^2 3x\, dx = \int \frac{1}{sin^2 3x cos^2 3x} dx$

$= \int \frac{4}{4sin^2 3x cos^2 3x} dx = 4\int \frac{1}{(2sin3x\, cos3x)^2} dx$

$= 4\int \frac{1}{sin^2 6x} dx = 4\int csc^2 6x\, dx = -\frac{2}{3}\, cot6x + c$

OR: $\int sec^2 3x csc^2 3x\, dx = \int \frac{1}{sin^2 3x cos^2 3x} dx$

$= \int \frac{sin^2 3x + cos^2 3x}{sin^2 3x cos^2 3x} dx = \int (sec^2 3x + csc^2 3x)$

$= \frac{1}{3} tan3x - \frac{1}{3} cot3x + c$

198) $\int \sin^7 x \cos x \, dx$

$= \frac{\sin^8 x}{8} + c$

....

199) $\int \frac{\sec x \tan x}{\sqrt{\sec x}} dx$

$= \int \frac{\sqrt{\sec x}\sqrt{\sec x}\ \tan x}{\sqrt{\sec x}} dx = \int \sqrt{\sec x} \tan x \, dx = \int \sec^{\frac{1}{2}} x \tan x \, dx$

$= \int \sec^{\frac{-1}{2}} x \sec x \tan x \, dx = 2\sec^{\frac{1}{2}} x + c = 2\sqrt{\sec x} + c$

....

200) $\int (\sec x \tan x)^4 dx$

$= \int sce^4 x \tan^4 x dx = \int \tan^4 x(\tan^2 x + 1)\sec^2 x dx$

$= \int (\tan^6 x \sec^2 x + \tan^4 x \sec^2 x) = \frac{1}{7}\tan^7 x + \frac{1}{5}\tan^5 x + c$

....

201) $\int \frac{\cot x}{\sin x} dx$

$= \int \csc x \cot x \, dx = -\csc x + c$

....

202) $\int \frac{\sec^6 x}{\cot x} dx$

$= \int \sec^6 x \tan x dx = \int \sec^5 x \sec x \tan x dx = \frac{1}{6}\sec^6 x + c$

203) $\int \cos^4x \sin2x\, dx$

$=\int \cos^4x(2\sin x \cos x)dx = 2\int \cos^5x \sin x\, dx = -2\frac{\cos^6x}{6} + c$

$=-\frac{1}{3}\cos^6x + c$

204) $\int \frac{\cos^4x-\sin^4x}{\sqrt{1+\sin2x}}dx$

$=\int \frac{(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)}{\sqrt{1+\sin2x}}dx = \int \frac{\cos2x}{\sqrt{1+\sin2x}}dx = \int(1+\sin2x)^{-\frac{1}{2}}\cos2x dx$

$=\frac{1}{2}.2\,(1+\sin2x)^{\frac{1}{2}} + c = \sqrt{1+\sin2x} + c$

OR: $\int \frac{\cos^4x-\sin^4x}{\sqrt{1+\sin2x}}dx = \int \frac{(\cos^2x-\sin^2x)(\cos^2x+\sin^2x)}{\sqrt{\cos^2x+\sin^2x+2\sin x\cos x}}dx$

$=\int \frac{(\cos x+\sin x)(\cos x-\sin x)}{\sqrt{(\cos x+\sin x)^2}}dx = \pm\int \frac{(\cos x+\sin x)(\cos x-\sin x)}{\cos x+\sin x}dx$

$=\pm\int(\cos x-\sin x)dx = \pm(\sin x+\cos x) + c$

---

205) $\int \sqrt{1+\cos x}dx$

$=\int \sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}dx$

$=\int \sqrt{2\cos^2\frac{x}{2}}dx = \pm\sqrt{2}\int \cos\frac{x}{2}dx = \pm2\sqrt{2}\sin\frac{x}{2} + c$

206) $\int \sqrt{1 - sinx}\, dx$

$= \int \sqrt{cos^2 \frac{x}{2} + sin^2 \frac{x}{2} - 2sin \frac{x}{2}\ cos \frac{x}{2}}\, dx$

$= \int \sqrt{\left(cos \frac{x}{2} - sin \frac{x}{2}\right)^2}\, dx = \pm \int \left(cos \frac{x}{2} - sin \frac{x}{2}\right) dx$

$= \pm 2 \left(sin \frac{x}{2} + cos \frac{x}{2}\right) + c$

---

207) $\int \sqrt{1 + sinx}\, dx$

$= \int \sqrt{cos^2 \frac{x}{2} + sin^2 \frac{x}{2} + 2sin \frac{x}{2}\ cos \frac{x}{2}}\, dx$

$= \int \sqrt{\left(cos \frac{x}{2} + sin \frac{x}{2}\right)^2}\, dx = \pm \int \left(cos \frac{x}{2} + sin \frac{x}{2}\right) dx$

$= \pm 2 \left(sin \frac{x}{2} - cos \frac{x}{2}\right) + c$

---

208) $\int \sqrt{1 - cosx}\, dx$

$= \int \sqrt{cos^2 \frac{x}{2} + sin^2 \frac{x}{2} - cos^2 \frac{x}{2} + sin^2 \frac{x}{2}}\, dx$

$= \int \sqrt{2sin^2 \frac{x}{2}}\, dx = \pm\sqrt{2} \int sin \frac{x}{2}\, dx = \pm 2\sqrt{2} cos \frac{x}{2} + c$

209) $\int \sqrt{1-\sin 3x}\,dx$

$=\int \sqrt{\cos^2\frac{3x}{2}+\sin^2\frac{3x}{2}-2\sin\frac{3x}{2}\,\cos\frac{3x}{2}}\,dx$

$=\int \sqrt{\left(\cos\frac{3x}{2}-\sin\frac{3x}{2}\right)^2}\,dx=\pm\int\left(\cos\frac{3x}{2}-\sin\frac{3x}{2}\right)dx$

$=\pm\frac{2}{3}\left(\sin\frac{3x}{2}+\cos\frac{3x}{2}\right)+c$

---

210) $\int \frac{\cot x+\tan x}{\cot x-\tan x}\,dx$

$=\int \frac{\frac{\cos x}{\sin x}+\frac{\sin x}{\cos x}}{\frac{\cos x}{\sin x}-\frac{\sin x}{\cos x}}\,dx=\int \frac{\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}}{\frac{\cos^2 x-\sin^2 x}{\sin x\cos x}}\,dx=\int \frac{\cos^2 x+\sin^2 x}{\cos^2 x-\sin^2 x}\,dx=$

$\int \frac{1}{\cos^2 x-\sin^2 x}\,dx$

$=\int \frac{1}{\cos 2x}\,dx=\int \sec 2x\,dx=\frac{1}{2}\ln|\sec 2x+\tan 2x|+c$

---

211) $\int \frac{\sec(\ln x)\tan(\ln x)}{x}\,dx$

$=\int \sec(\ln x)\tan(\ln x).\frac{1}{x}\,dx$

$=\sec(\ln x)+c$

212) $\int \frac{csc(lnx^3)cot(lnx^3)}{x}dx$

$=\int csc(lnx^3)cot\,(lnx^3).\frac{1}{x}dx$

$=-\frac{1}{3}csc(lnx^3)+c$

U=(lnx^3)

Du =$\frac{1}{x^3}.3x^2$=$\frac{3}{x}$

213) $\int(cscx+cotx)dx$

$=\int\left(cscx.\frac{cscx+cotx}{cscx+cotx}+\frac{cosx}{sinx}\right)dx=\int\left(\frac{csc^2x+cscx\,cotx}{cscx+cotx}+\frac{cosx}{sinx}\right)dx$

$=-ln|cscx+cotx|+ln|sinx|+c$

$=ln\left|\frac{sinx}{cscx+cotx}\right|+c$

214) $\int\frac{10+15cos2x+5sin3x+cos5x}{32}dx$

$=\int\left(\frac{10}{32}+\frac{15}{32}cos2x+\frac{5}{32}sin3x+\frac{1}{32}cos5x\right)dx$

$=\frac{10}{32}x+\frac{15}{64}sin2x-\frac{5}{96}cos3x+\frac{1}{160}sin5x+c$

215) $\int(1+cos^23x)dx$

$=\int\left(1+\frac{1}{2}+\frac{1}{2}cos6x\right)dx=\int\left(\frac{3}{2}+\frac{1}{2}cos6x\right)dx$

$=\frac{3}{2}x+\frac{1}{12}sin6x+c$

216) $\int (cos^4 x - sin^4 x)dx$

$=\int (cos^2 x - sin^2 x)(cos^2 x + sin^2 x)dx$

$=\int cos2xdx = \frac{1}{2} sin2x + c$

....

217) $\int (1 + cos3x)^2 dx$

$=\int (1 + 2cos3x + cos^2 3x)dx = \int \left(1 + 2cos3x + \frac{1}{2} + \frac{1}{2} cos6x\right) dx$

$=\int \left(\frac{3}{2} + 2cos3x + \frac{1}{2} cos6x\right) dx = \frac{3}{2} x + \frac{2}{3} sin3x + \frac{1}{12} sin6x + c$

....

218) $\int sin^2 x^2 cosx^2 xdx$

$=\frac{1}{2} . \frac{1}{3} \quad = \frac{1}{6} \ sin^3 x^2 + c$

....

219) $\int \frac{5}{csc^2 x\sqrt{tanx-5}} dx$

$=5 \int (tanx - 5)^{\frac{-1}{2}} sec^2 xdx = 5.2(tanx - 5)^{\frac{1}{2}} + c$

$=10\sqrt{tanx - 5} + c$

220) $\int (sin2x-1)\left(cos^2 2x+2\right)dx$

$=\int (cos^2 2xsin2x+2sin2x-cos^2 2x-2)dx$

$=\int \left(cos^2 2x\, sin2x+2sin2x-\frac{1}{2}-\frac{1}{2}cos4x-2\right)dx$

$=\int \left(cos^2 2x\, sin2x+2sin2x-\frac{1}{2}cos4x-\frac{5}{2}\right)dx$

$=-\frac{1}{6}cos^3 2x-cos2x-\frac{1}{8}sin4x-\frac{5}{2}x+c$

....

221) $\int \frac{2+sinx-sin^2x+sin^3x}{cos^2x}dx$

$=\int \frac{2+sinx-sin^2x+sinx-sinxcos^2x}{cos^2x}dx=\int \frac{2+2sinx-sin^2x-sinxcos^2x}{cos^2x}dx$

$=\int (2sec^2x+2secxtanx-tan^2x-sinx)dx$

$=\int (2sec^2x+2secxtanx-sec^2x+1-sinx)dx$

$=\int (sec^2x+2secxtanx+1-sinx)dx$

$=tanx+2secx+x+cosx+c$

....

222) $\int sin\left(\frac{x-1}{x+1}\right)\left(\frac{3}{x+1}\right)^2dx$

$=9\int sin\left(\frac{x-1}{x+1}\right)\frac{1}{(x+1)^2}dx=-\frac{9}{2}cos\left(\frac{x-1}{x+1}\right)+c$

....

223) $\int \frac{sec^2(sinx)}{secx}dx$

$=\int sec^2(sinx)cosxdx=tan(sinx)+c$

224) $\int\left(\frac{sinx}{1-cosx}+cotx\right)dx$

$=\int\left(\frac{sinx}{1-cosx}\frac{1+cosx}{1+cosx}+\frac{cosx}{sinx}\right)dx=\int\left(\frac{sinx(1+cosx)}{1-cos^2x}+\frac{cosx}{sinx}\right)dx$

$=\int\left(\frac{sinx(1+cosx)}{sin^2x}+\frac{cosx}{sinx}\right)dx=\int\left(\frac{1+2cosx}{sinx}\right)dx=\int\left(cscx+2\frac{cosx}{sinx}\right)dx$

$=-ln|cscx+cotx|+2ln|sinx|+c=ln\left|\frac{sin^2x}{cscx+cotx}\right|+c$

225) $\int\frac{\sqrt{1+sinx}}{sinx}dx$

$=\int\frac{\sqrt{sin^2\frac{x}{2}+2sin\frac{x}{2}cos\frac{x}{2}+cos^2\frac{x}{2}}}{sinx}dx=\int\frac{\sqrt{\left(sin\frac{x}{2}+cos\frac{x}{2}\right)^2}}{sinx}dx$

$=\pm\int\frac{sin\frac{x}{2}+cos\frac{x}{2}}{2sin\frac{x}{2}cos\frac{x}{2}}dx=\pm\frac{1}{2}\int\left(\frac{1}{cos\frac{x}{2}}+\frac{1}{sin\frac{x}{2}}\right)dx=\pm\frac{1}{2}\int\left(sec\frac{x}{2}+csc\frac{x}{2}\right)dx$

$=\pm\left[ln\left|sec\frac{x}{2}+tan\frac{x}{2}\right|-ln\left|csc\frac{x}{2}+cot\frac{x}{2}\right|\right]+c$

226) $\int\frac{1-cos2x}{tanx}dx$

$=\int\frac{1-1+2sin^2x}{tanx}dx=\int 2sin^2x.cotxdx=2\int sin^2x.\frac{cosx}{sinx}dx$

$=2\int sinx\,cosxdx=2.\frac{1}{2}sin^2x+c=sin^2x+c$

227) $\int \frac{1-\cos 2x}{1-\sin 2x}dx$

$=\int \frac{1-\cos 2x}{1-\sin 2x}\cdot\frac{1+\sin 2x}{1+\sin 2x}dx = \int \frac{1+\sin 2x-\cos 2x-\cos 2x\sin 2x}{1-\sin^2 x}dx$

$=\int \frac{1+\sin 2x-\cos 2x-\cos 2x\sin 2x}{\cos^2 2x}dx$

$=\int \left(\sec^2 2x+\sec 2x\tan 2x-\sec 2x-\frac{\sin 2x}{\cos 2x}\right)dx$

$=\frac{1}{2}[\tan 2x+\sec 2x-\ln|\sec 2x+\tan 2x|+\ln|\cos 2x|]+c$

228) $\int \left(\frac{\cos x}{1-\sin x}-\frac{\cos x}{1+\sin x}\right)dx$

$=\int \frac{\cos x+\cos x\sin x-\cos x+\cos x\sin x}{1-\sin^2 x}dx$

$=\int \frac{2\cos x\sin x}{\cos^2 x}dx = 2\int \frac{\sin x}{\cos x}dx = -2\ln|\cos x|+c$

---

229) $\int \left(\frac{\cos x}{1-\sin x}+\frac{\cos x}{1+\sin x}\right)dx$

$=\int \frac{\cos x+\cos x\sin x+\cos x-\cos x\sin x}{1-\sin^2 x}dx$

$=\int \frac{2\cos x}{\cos^2 x}dx = 2\int \frac{1}{\cos x}dx = 2\int \sec x\,dx = 2\ln|\sec x+\tan x|+c$

230) $\int \frac{1}{1+\sec x}dx$

$=\int \frac{1}{1+\frac{1}{\cos x}}dx = \int \frac{\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}dx$

$=\int \frac{\cos x-\cos^2 x}{\sin^2 x}dx = \int(\csc x\cot x-\cot^2 x)dx$

$=\int(\csc x\cot x-\csc^2 x+1)dx$

$=-\csc x+\cot x+x+c$

231) $\int \frac{1}{1+cscx}dx$

$=\int \frac{sinx}{1+sinx}\cdot\frac{1-sinx}{1-sinx}dx = \int \frac{sinx-sin^2x}{1-sin^2x}dx = \int \frac{sinx-sin^2x}{cos^2x}dx$

$=\int (secx\,tanx - tan^2x)dx$

$=\int (secx\,tanx - sec^2x + 1)dx = secx - tanx + x + c$

---

232) $\int \left(1 - \frac{cos^2x}{1+sinx}\right)dx$

$=\int \left(1 - \frac{(1-sinx)(1+sinx)}{1+sinx}\right)dx = \int (1 - 1 + sinx)dx$

$=\int sinxdx = -cosx + c$

---

233) $\int \frac{cot^2x}{1+sinx}dx$

$=\int \frac{cot^2x}{1+sinx}\cdot\frac{1-sinx}{1-sinx}dx = \int \frac{cot^2x - sinxcot^2x}{1-sin^2x}dx$

$=\int \left(\frac{cos^2x}{sin^2x}\cdot\frac{1}{cos^2x} - \frac{sinx}{cos^2x}\cdot\frac{cos^2x}{sin^2x}\right)dx = \int (csc^2x - cscx)dx$

$=-cotx + ln|cscx + cotx| + c$

---

234) $\int \left(\frac{cosx}{1+sinx} + \frac{1+sinx}{cosx}\right)dx$

$=\int \left(\frac{cosx}{1+sinx}\cdot\frac{1-sinx}{1-sinx} + \frac{1+sinx}{cosx}\right)dx = \int \left(\frac{cosx(1-sinx)}{cos^2x} + \frac{1+sinx}{cosx}\right)dx$

$=\int \frac{1-sinx+1+sinx}{cosx}dx = \int \frac{2}{cosx}dx = 2\int secxdx$

$=2ln|secx + tanx| + c$

235) $\int \frac{cos2x}{1-cos4x}dx$

$=\int \frac{cos2x}{1-1+2sin^2 2x}dx = \int \frac{cos2x}{2sin^2 2x}dx$

$=\frac{1}{2}\int csc2xcot2xdx = -\frac{1}{4}csc2x + c$

---

236) $\int \frac{1}{1+tanx}dx$

$=\int \frac{1}{1+\frac{sinx}{cosx}}dx = \int \frac{cosx}{sinx+cosx}dx = \frac{1}{2}\int \frac{2cosx}{sinx+cosx}dx$

$=\frac{1}{2}\int \frac{2cosx+sinx-sinx}{sinx+cosx}dx = \frac{1}{2}\int \frac{cosx+sinx}{sinx+cosx}dx + \frac{1}{2}\int \frac{cosx-sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int dx + \frac{1}{2}\int \frac{cosx-sinx}{sinx+cosx}dx = \frac{1}{2}x + ln|sinx + cosx| + c$

237) $\int \frac{1}{1+cotx}dx$

$=\int \frac{1}{1+\frac{cosx}{sinx}}dx = \int \frac{sinx}{sinx+cosx}dx = \frac{1}{2}\int \frac{2sinx}{sinx+cosx}dx$

$=\frac{1}{2}\int \frac{2sinx+cosx-cosx}{sinx+cosx}dx = \frac{1}{2}\int \frac{sinx+cosx}{sinx+cosx}dx + \frac{1}{2}\int \frac{sinx-cosx}{sinx+cosx}dx$

$= \frac{1}{2}\int dx - \frac{1}{2}\int \frac{cosx-sinx}{sinx+cosx}dx = \frac{1}{2}x - \frac{1}{2}ln|sinx + cosx| + c$

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٢٣٨) $\int \sqrt{1 + sinx}\, d(sinx)$

$=\int (1 + sinx)^{\frac{1}{2}}cosxdx = \frac{2}{3}(1 + sinx)^{\frac{3}{2}} + c = \frac{2}{3}\sqrt{(1 + sinx)^3} + c$